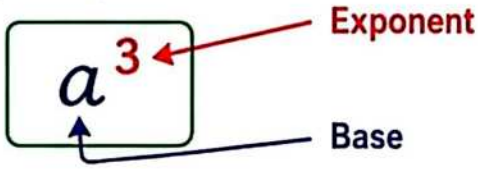
LESSON 1: POWER OF A POWER

1 INTRODUCTION

What is the Power (Exponent)?

The power (or exponent) is the number that shows how many times the base is multiplied by itself.



What does a power (exponent) mean?

The power shows how many times the base is multiplied by itself.

Example:

$$a^3 = a \times a \times a$$

2 THE LAW OF POWER OF A POWER

If we raise a power to another power, we multiply the exponents and keep the same base.

$$(a^m)^n = a^{m \times n}$$

(The base remains the same, only the exponents are multiplied. ☺)

3 EXAMPLES ★

Choose the correct answer:

③

$$\frac{4^{a+2} \times 9^{3+a}}{6^{2a+3}}$$

$$= \frac{(2^2)^{a+2} \times (3^2)^{3+a}}{(2 \times 3)^{2a+3}}$$

$$= \frac{2^{2(a+2)} \times 3^{2(3+a)}}{2^{2a+3} \times 3^{2a+3}}$$

$$= 2^{2a+4-(2a+3)} \times 3^{6+2a-(2a+3)}$$

$$= 2^1 \times 3^3 = 2 \times 27$$

$$= 54$$

②

$$\frac{(a^2)^{-3} \times (a^{-1})^2}{a^{-3} \times a^{-4}}$$

$$= \frac{a^{-6} \times a^{-2}}{a^{-3} \times a^{-4}}$$

$$= \frac{a^{-6+(-2)}}{a^{-3+(-4)}} = \frac{a^{-8}}{a^{-7}}$$

$$= a^{-8-(-7)} = a^{-1}$$

$$= \frac{1}{a}$$

①

$$\frac{(2^{11})^2 \times 2^3}{(2^6)^3 \times 2^4}$$

$$= \frac{2^{22} \times 2^3}{2^{18} \times 2^4}$$

$$= \frac{2^{22+3}}{2^{18+4}}$$

$$= \frac{2^{25}}{2^{22}} = 2^{25-22}$$

$$= 2^3 = 8$$

PRACTICE FOR YOU

① $\dfrac{((\sqrt{5})^3)^2 \times \sqrt{5}}{(-\sqrt{5})^5}$

② $\dfrac{a^4 \times (a^{-1})^4}{(a^2)^5 \times a^{-2}}$

③ $\dfrac{36^n \times 5^{2n}}{30^{2n}}$

4 NOTES

When there is a power of a power, we multiply the exponents only.

★ The base must be the same, otherwise we cannot do this.

★ The exponent is distributed over multiplication or division, but not over addition or subtraction.

Examples:

$(a+b)^n \neq a^n + b^n$ ✗ $(a-b)^n \neq a^n - b^n$ ✗

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EXPONENTIAL EQUATIONS

1 WHAT IS AN EXPONENTIAL EQUATION?

Definition:

An exponential equation is an equation that contains a variable in the exponent.

Examples of NOT exponential equations ✗	Examples of exponential equations ✓
• $x^2 + 5 = 9$	• $2^x = 16$
• $3x - 2 = 7$	• $3^{x+1} = 27$
• $2x^2 - 1 = 0$	• $5^{x-2} = \frac{1}{25}$

The variable (x) is in the exponent.

2 HOW TO SOLVE EXPONENTIAL EQUATIONS

Main idea:

If the bases are equal (or can be made equal), we equate the exponents...

Then we solve for the unknown.

✓ Bases are equal

Then solve the equation.

THE LAW

If $a^m = a^n$ where $a > 0, a \neq 1$

then $m = n$

3 POWER REFERENCE TABLE

Memorize the powers

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$	$7^1 = 7$	$10^1 = 10$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$	$7^2 = 49$	$10^2 = 100$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$	$7^3 = 343$	$10^3 = 1000$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$	$7^4 = 2401$	
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$		
$2^6 = 64$	$3^6 = 729$			

Memorizing powers makes solving exponential equations much easier!

4 SOLVED EXAMPLES

Choose the correct equation and solve it:

① $2^{x+3} = 64$

Solution:

$64 = 2^6$

$2^{x+3} = 2^6$

$x + 3 = 6$

$x = 3$

② $3^{2x-1} = 243$

Solution:

$243 = 3^5$

$3^{2x-1} = 3^5$

$2x - 1 = 5$

$2x = 6$

$x = 3$

③ $5^{x-2} = \frac{1}{125}$

Solution:

$\frac{1}{125} = 5^{-3}$

$5^{x-2} = 5^{-3}$

$x - 2 = -3$

$x = -1$

5 PRACTICE FOR YOU

Solve the following equations:

① $2^{x-1} = 32$

$x =$ _____

② $3^{x+2} = 81$

$x =$ _____

③ $7^{2x-1} = 343$

$x =$ _____

STEPS TO SOLVE (in 3 steps)

1. Identify the base (make sure it's the same on both sides).
2. Equate the exponents.
3. Solve the equation to find the value of the variable.

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Mastering Exponents

9 Identify the Mistake:

Asmaa's Solution

$(a^2)^3 \times a^4 = a^6 \times a^4 = a^{10}$

Khalid's Solution

$(a^2)^3 \times a^4 = a^5 \times a^4 = a^{20}$

Khalid and Asmaa solved $(a^2)^3 \times a^4$ in two different ways.

Whose solution is correct? Explain why.

Asmaa ✓ Correct

Khalid ✗ Incorrect

Reason:

$(a^2)^3 \times a^4$

$= (a^6) \times a^4$

$= a^{6+4} = a^{10}$

10 Apply the Concepts

Simplify the following expressions.

1 $(a^3)^4 \times a^5$

$= a^{12} \times a^5$

$= a^{12+5} = a^{17}$

2 $(a^2)^5 \times (a^4)^3$

$= a^{10} \times a^{12}$

$= a^{10+12} = a^{22}$

3 $(\sqrt{3})^2 \times (-\sqrt{3})^4$

$= (\sqrt{3})^2 \times (\sqrt{3})^4$

$= (\sqrt{3})^6$

$= (\sqrt{3})^{2\times 3} = (\sqrt{3})^6$

4 $\dfrac{(a^5)^2}{(a^3)^4}$

$= \dfrac{a^{10}}{a^{12}}$

$= a^{10-12} = a^{-2}$

5 $\dfrac{(a^2)^4}{a^5}$

$= \dfrac{a^8}{a^5}$

$= a^{8-5} = a^3$

6 $((-3)^4)^2 \times (-3)^{-3}$

$= (-3)^8 \times (-3)^{-3}$

$= (-3)^{8+(-3)}$

$= (-3)^5 = -243$

7 $((-\sqrt{2})^3)^{-2}$

$= (-\sqrt{2})^{-6}$

$= (-1)^6 \times (\sqrt{2})^{-6}$

$= (\sqrt{2})^{-6}$

$= \dfrac{1}{(\sqrt{2})^6} = \dfrac{1}{2^3} = \dfrac{1}{8}$

8 $\dfrac{((\sqrt{5})^2)^3 \times (\sqrt{5})^{-1}}{(\sqrt{5})^4}$

$= \dfrac{(\sqrt{5})^{2\times 3} \times (\sqrt{5})^{-1}}{(\sqrt{5})^4}$

$= \dfrac{(\sqrt{5})^{6-1}}{(\sqrt{5})^4}$

$= (\sqrt{5})^{5-4} = (\sqrt{5})^1 = \sqrt{5}$

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Remember:
When multiplying same bases, add the exponents.

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Choose the correct simplified form:

1 $(5a^3b^2c)^2$

$= (5)^2 (a^3)^2 (b^2)^2 (c)^2$

$= 25\,a^6\,b^4\,c^2$

$= \boxed{25a^6b^4c^2}$

2 $(-2x^4y^2)^3$

$= (-2)^3 (x^4)^3 (y^2)^3$

$= -8\,x^{12}\,y^6$

$= \boxed{-8x^{12}\,y^6}$

3 $\left(\dfrac{3p^2q^{-1}}{2p^3q}\right)^{-2}$

$= \left(\dfrac{3p^2q^{-1}}{2p^3q}\right)^{-2}$

$= \left(\dfrac{3}{2}\,p^{-1}q^{-2}\right)^{-2}$

$= \left(\dfrac{2}{3}\right)^2 p^2 q^4$

$= \boxed{\dfrac{4}{9}\,p^2q^4}$

4 $(d^4)^2 \times (-3d^2)^3$

$= d^8 \times (-3)^3\,d^6$

$= d^8 \times (-27)\,d^6$

$= -27\,d^{14}$

$= \boxed{-27\,d^{14}}$

5 $(2x^2y)^3 \times (-xy^2)^2$

$= (2^3x^6y^3) \times ((-1)^2x^2y^4)$

$= 8\,x^6y^3 \times x^2\,y^4$

$= 8\,x^8y^7$

$= \boxed{8\,x^8y^7}$

6 $\dfrac{4\,m\,p^{-3}}{(m^{-1}p^2)^2}$

$= \dfrac{4\,m\,p^{-3}}{m^{-2}\,p^4}$

$= 4\,m^{1-(-2)}\,p^{-3-4}$

$= \boxed{4\,m^3\,p^{-7}}$

7 $(f^2g^{-1}h^3)^{-2}$

$= f^{-4}\,g^2\,h^{-6}$

$= \dfrac{1}{f^4} \times g^2 \times \dfrac{1}{h^6}$

$= \boxed{\dfrac{g^2}{f^4\,h^6}}$

8 $\dfrac{(9a^2b^{-3})^0}{(3a^0b^{-2})^{-2}}$

$= \dfrac{1}{(3 \cdot 1 \cdot b^{-2})^{-2}}$

$= \dfrac{1}{3^{-2}\,b^4}$

$= 3^2\,b^{-4}$

$= \boxed{\dfrac{9}{b^4}}$

9 $\dfrac{(3a^3)^2 \times (2a^2)^4}{(6a^4)^3 \times (a^{-1})^2}$

$= \dfrac{3^2\,a^6 \times 2^4\,a^8}{6^3\,a^{12} \times a^{-2}}$

$= \dfrac{9 \times 16 \times a^{14}}{216\ a^{10}}$

$= \dfrac{144}{216}\,a^4$

$= \boxed{\dfrac{2}{3}\,a^4}$

Exponent Rules Reminder

- $(a^m)^n = a^{mn}$
- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n} \quad (b \neq 0)$
- $a^{-n} = \dfrac{1}{a^n} \quad (a \neq 0)$

Tip

When multiplying same bases, add the exponents. When dividing, subtract the exponents.

1. Simplify to simplest form, find value at $x = 1$ (1)

(1)

$$\frac{4^x \times 2^{x-4}}{16^{x-2}}$$

$$\Rightarrow \frac{2^{2x} \times 2^{x-4}}{2^{4(x-2)}}$$

$$\Rightarrow \frac{2^{3x-4}}{2^{4x-8}}$$

$$\Rightarrow \frac{2^{3x+4}}{2^{4x-8}} = 2^{-x+4}$$

At x = 1 : $2^{-1+4} = 8$

(2)

$$\frac{4^x \times (49)^{x-1}}{(98)^x}$$

$$\Rightarrow \frac{2^{2x} \times 7^{2x-2}}{(49 \times 2)^x}$$

$$\Rightarrow \frac{2^{2x} \times 7^{2x-2}}{7^{2x} \times 2^x}$$

$$\Rightarrow \frac{2^{2x}}{2^x} \times \frac{7^{2x-2}}{7^{2x}}$$

$$\Rightarrow 2^x \times 7^{-2} = \frac{2^x}{49}$$

At x = 1 : $= \frac{2}{49}$

14. If $3^a = 2$, find the value of: (14)

$$\Rightarrow 9^a + 27^a$$

$$\Rightarrow (3^2)^a + (3^3)^a$$

$$\Rightarrow 3^{2a} + 3^{3a}$$

$$\Rightarrow (3^a)^2 + (3^a)^3$$

$$\Rightarrow 2^2 + 2^3$$

$$\Rightarrow 4 + 8 = 12$$

13. Prove that: $\frac{27^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \frac{1}{27}$ (13)

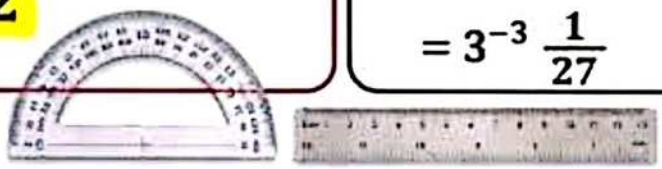
$$\text{LHS} = \frac{(3^3)^{x-1} \times (2^3)^x}{2^{2x} \times (\sqrt{2})^{2x} \times 3^{2x} \times (\sqrt{3})^{2x}}$$

$$= \frac{3^{3x-3} \times 2^{3x}}{(2^2)^x \times 2^x \times 3^{2x} \times 3^x}$$

$$= \frac{3^{3x-3}}{2^{3x}} \times \frac{2^{3x}}{3^{3x}}$$

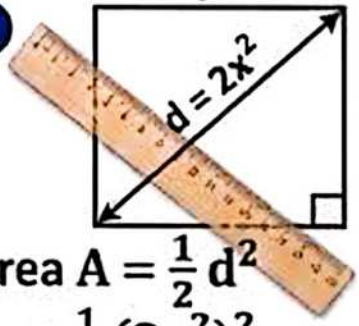
$$= 3^{3x-3-3x} = \frac{1}{3^3} = \frac{1}{27} = \text{RHS}$$

$$= 3^{-3}\ \frac{1}{27}$$



15. Geometry: Find area of each shape in simplest form: (15)

(1)

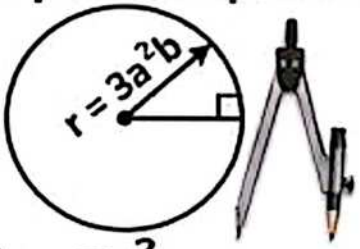


Area $A = \frac{1}{2} d^2$

$= \frac{1}{2}(2x^2)^2$

$= \frac{1}{2}(4x^4) = 2x^4\ \text{cm}^2$

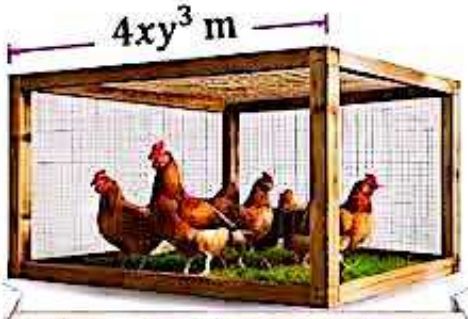
(2)



Area $A = \pi r^2$

$= \pi (3a^2b)^2$

$= \pi \times 9a^4b^2 = 9\pi a^4b^2\ \text{cm}^2$



A cube-shaped chicken coop has an edge length of $4xy^3$ meters.

Cube Volume 16

Poultry is an important livestock resource, a key protein source.

$V = s^3$

$V = (4xy^3)^3$

$= 64\,x^3y^9$

$= 64x^3y^9$ meters^3

17 A cylindrical barrel for fuel storage has a base radius of $2kw^2$ meters and a height of $4k$ meters. Find the barrel's volume in simplest form.

$V = \pi r^2 \times h$

$= \pi\,(2kw^2)^2 \times 4k$

$= \pi\,(4k^2w^4) \times 4k$

$= 16\pi\,k^3w^4$ meters^3

Sustainable fuel consumption helps conserve material resources and protect the environment.

Creative Thinking 18

$(2 \times 5)^x = 5^{x+1}$

$2^x \times 5^x = 5^x \times 5$

$4^x = 5$

If $10^x = 5^{x+1}$, find the value of 4^x.

$\therefore\ 4^x = (2^x)^2 = 25$ result = 25

19 If $a = 5^{30}$, $b = 3^{45}$, $c = 2^{60}$, arrange a, b, c in ascending order.

$\therefore\ c = (2^4)^{15}$ ، $b = (3^3)^{15}$ ، $a = (5^2)^{15}$

$c = (16)^{15}$ ، $b = (27)^{15}$ ، $a = (25)^{15}$

$16 < 25 < 27 \Rightarrow \therefore\ c < a < b$

2027
3rd Preparatory Grade

MATH ESSENTIALS

$x^n = x^m$

LESSON EVALUATION

PART 1 : Exponential Equations

Lesson Evaluation أولا

Choose the correct answer from the given choices:

If: $2^x = 8$, then what is the value of x? (2)

a) 1 b) 2 **c) 3 ✓** d) 4

$2^x = 8$

$2^x = 2^3$

$\therefore x = 3$

If: $2^{x-3} = 1$, then what is the value of x? (1)

a) 0 b) 1 c) 2 **d) 3 ✓**

$2^{x-3} = 1$

$2^{x-3} = 2^0$

$x - 3 = 0 \Rightarrow x = 3$

If: $3^{x+2} = 3^{2x-1}$, then what is the value of x? (4)

a) 2 **b) 3 ✓** c) 4 d) 5

$3^{x+2} = 3^{2x-1}$

$x + 2 = 2x - 1$

$2 + 1 = 2x - x$

$3 = x$

$\therefore x = 3$

If: $3^{-x} = 3$, then what is the value of x? (3)

a) −3 **b) −1 ✓** c) 1 d) 3

$3^{-x} = 3^1$

$-x = 1$

$\therefore x = -1$

If: $27 = 9^x$, then what is the value of x? (6)

a) $\frac{2}{3}$ **b) $\frac{3}{2}$ ✓** c) $\frac{1}{3}$ d) 3

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$\therefore x = \frac{3}{2}$

If: $3^{x-1} = 5^{x-1}$, then what is the value of x? (5)

a) 0 **b) 1 ✓** c) 3 d) 5

$3^{x-1} = 5^{x-1}$

$\left(\frac{3}{5}\right)^{x-1} = 1 = \left(\frac{3}{5}\right)^0$

$x - 1 = 0$

$\therefore x = 1$

If: $3^x + 3^x = 6$, then what is the value of x? (8)

a) 1 ✓ b) 2 c) 3 d) 9

$3^x + 3^x = 6$

$2 \times 3^x = 6$

$3^x = 3^1$

$\therefore x = 1$

Recall:

$2 \times 3 = 6$

$\therefore 3^1 = 3$

$\Rightarrow x = 1$

If: $5^{2x-1} = \left(\frac{1}{5}\right)^{-7}$, then what is the value of x? (7)

a) 2 b) 3 **c) 4 ✓** d) 5

$5^{2x-1} = (5^{-1})^{-7} = 5^7$

$2x - 1 = 7$

$2x = 8$

$\therefore x = 4$

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Part Two Concept Application

9 Solve each of the following equations in $\mathbb{R}$:

3 $4^{1-2x} = 2$

$4^{1-2x} = 2$

$2^{2(1-2x)} = 2$

$= 2$

$2 - 4x = 1$

$-4x = -1$

$x = \frac{1}{4}$

2 $3^{2x-3} = \frac{1}{27}$

$3^{2x-3} = (3)^{-3}$

$\Rightarrow 2x - 3 = -3$

$2x = 0$

$x = 0$

1 $5^{2x-1} = 125$

$5^{2x-1} = 5^3$

$\Rightarrow 2x - 1 = 3$

$2x = 4$

$x = 2$

6 $\left(\frac{3}{5}\right)^{2x+1} = \frac{125}{27}$

$\left(\frac{3}{5}\right)^{2x+1} = \left(\frac{3}{5}\right)^{-3}$

$2x + 1 = -3$

$2x = -4$

$x = -2$

5 $\left(\frac{2}{5}\right)^{2x+1} = \frac{8}{125}$

$\left(\frac{2}{5}\right)^{2x+1} = \left(\frac{2}{5}\right)^3$

$2x + 1 = 3$

$2x = 2$

$x = 1$

4 $\left(\frac{1}{8}\right)^{x-2} = 128$

$2^{-3(x-2)} = 2^7$

$-3x + 6 = 7$

$-3x = 1$

$x = -\frac{1}{3}$

9 $9 \times 5^{x-1} = 25 \times 3^{x-1}$

$\frac{9}{25} = \left(\frac{3}{5}\right)^{x-1}$

$\left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)^{x-1}$

$x - 1 = 2$

$x = 3$

8 $7^{x+2} = 2^x \times 2^{-x}$

$7^{x+2} = 2^x \times 2^{-x}$

$7^{x+2} = 2^0 = 1$

$x + 2 = 0$

$x = -2$

7 $7^{2x+3} = 7^{x-4}$

$2x + 3 = x - 4$

$x = -7$

10 Find the solution set in $\mathbb{R}$ for each of the following equations:

3 $32^{x-3} = 8^{2x+1}$

$5x - 15 = 6x + 3$

$2 = 2$

$5x - 15 = 6x + 3$

$x = -18$

2 $\left(\frac{1}{2}\right)^{4x-2} = 8^{x+3}$

$\left(\frac{1}{2}\right)^{4x-2} = 2^{3x+9}$

$-4x + 2 = 3x + 9$

$-7x = 7$

$x = -1$

1 $(216)^x = \frac{1}{6^3}$

$3x = -3$

$x = -1$

④ $\dfrac{36^{x+1}}{6^{x-1}} = \dfrac{1}{216}$

Solution

$\dfrac{(6^2)^{x+1}}{6^{x-1}} = \dfrac{1}{6^3}$

$\dfrac{6^{2x+2}}{6^{x-1}} = 6^{-3}$

$6^{2x+2-(x-1)} = 6^{-3}$

$6^{x+3} = 6^{-3}$

$x + 3 = -3$

$\boxed{x = -6}$

⑤ $8^{x-1} \times 4^{2x+1} = 64$

Solution

$(2^3)^{x-1} \times (2^2)^{2x+1} = 2^6$

$2^{3x-3} \times 2^{4x+2} = 2^6$

$2^{7x-1} = 2^6$

$7x - 1 = 6$

$7x = 7$

$\boxed{x = 1}$

⑥ $\dfrac{8^x \times 9^x}{18^x} = 64$

Solution

$\dfrac{(8\times9)^x}{18^x} = 64$

$\dfrac{72^x}{18^x} = 64$

$\left(\dfrac{72}{18}\right)^x = 64$

$4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

⑪ If $6^{2n} = \dfrac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, find the value of n.

Solution

$2^{2n} = \dfrac{(7^2)^n \times (5^2)^{2n} \times (3)^{4n}}{7^{-n} \times (3\times5)^{4n}} = 7^3$

$\dfrac{7^{2n} \times 5^{4n} \times 3^{4n}}{7^{-n} \times 3^{4n} \times 5^{4n}} = 7^3$

$7^{2n-(-n)} = 7^3$

$7^{3n} = 7^3$

$3n = 3$

$\boxed{n = 1}$

Therefore:

$6^{2n} = 6^2 = 36$

Solution 1 (by substitution)

$16^{x+1} = 64^{x-1}$

$(4^2)^{x+1} = (4^3)^{x-1}$

$4^{2x+2} = 4^{3x-3}$

$2x + 2 = 3x - 3$

$3x - 2x = 2 + 3$

$x = 5$

Correct Answer ✓

Solution 2 (by powers)

$16^{x+1} = 64^{x-1}$

$(2^4)^{x+1} = (2^6)^{x-1}$

$2^{4x+4} = 2^{6x-6}$

$4x + 4 = 6x - 6$

$4 + 6 = 6x - 4x$

$10 = 2x \Rightarrow x = 5$

Correct Answer ✓

⑫ **Check:**

We tested both methods in the equation:

$16^{x+1} = 64^{x-1}$

Which method is correct in solving?

Both methods are correct and the correct solution is:

$x = 5$

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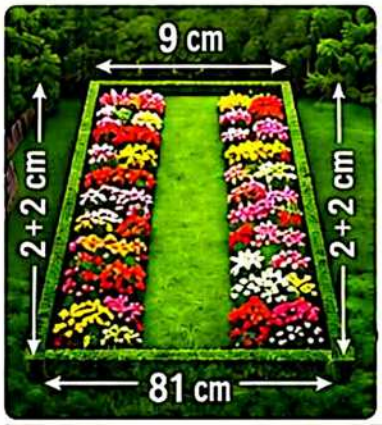
14 Geometry



A flowerbed has a rectangular shape as shown in the figure.

Find the perimeter of the flowerbed.

This flowerbed is surrounded by a path. Its width reflects taste and beauty.

$$9^{y} = 9^{2y-2}$$

$$\therefore y = 2$$

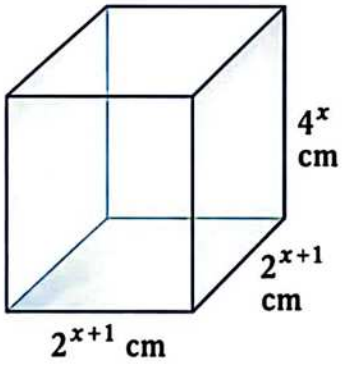
$$2^{4x} = 2^{3x+2}$$

$$4x = 3x + 2 \quad \therefore x = 2$$

$$256 = 4^{4} = \text{length}$$

$$\text{674 cm} = 2(256 + 81) \times 2 = 674$$

Analysis and Synthesis of Materials 13



Given:

A square box (cube) with edge length:

2^{x+1} cm, 4^{x} cm

Find the volume of one cube in terms of x.

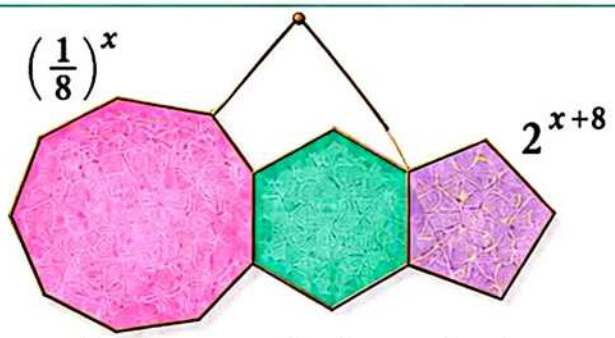
Volume of the cube:

$$S = 2^{x+1} \times 4^{x}$$

$$2^{x+1} = 2^{2x}$$

$$x + 1 = 2x \quad \therefore x = 1$$

$$\text{Volume} = 64 \text{ cm}^3 = (4)^3$$



Regular shapes with more sides are more beautiful and more functional.

15 Challenge:

In the figure shown,
a regular shape is hung on one side of a board.
The board consists of three regular shapes:
a regular octagon, a regular hexagon,
and a regular pentagon.
Find the value of x.

From the figure, we have:

$$\left(\frac{1}{8}\right)^{x} = 2^{x+8} \Rightarrow \left(\frac{1}{2}\right)^{3x} = 2^{x+8}$$

$$2^{-3x} = 2^{x+8} \Rightarrow -3x = x + 8$$

$$-4x = 8$$

$$x = \frac{-8}{-4}$$

$$x = -2$$

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CREATIVE THINKING

16 Find the value of x in each of the following:

$$2^x + 2^{x+1} + 2^{x+2} = 112 \quad (1)$$

- $2^x\,(1 + 2 + 4) = 112$
- $2^x \times \cancel{7} = 11\cancel{2}$
- $2^x \times \cancel{7} = 2^4 \times \cancel{7}$
- $2^x = 2^4$

$\therefore\ x = 4$

$$2 \times 3^{x+1} - 3^x = 5^{x+1} - 2 \times 5^x \quad (2)$$

- $3^x\,(2 \times 3 - 1) = 5^x\,(5 - 2)$
- $3^x \times 5 = 5^x \times 3$
- $\frac{3^x}{5^x} = \frac{3}{5}$
- $\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^1$

$x = 1$

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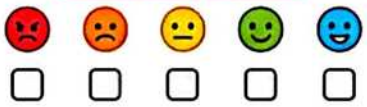
GRADE 9 – FIRST UNIT TEST 2027

How did you do?



How did you do on the first unit? Put a mark in the appropriate box.

Choose the correct answer from the given choices:

2 If $\frac{3^x \times 27^x}{81^x} = 2^{-2x}$, then what is the value of x?

(a) -1 (b) 1 (c) 2 (d) 0 ✓

$\frac{3^x \times 3^{3x}}{3^{4x}} = 2^{-2x}$ $\quad \therefore -2x = 0$

$2^{-2x} = 1$

$x = 0$

1 Which of the following is equal to $(a^2)^4 \times a^5$?

(a) a^{11} (b) a^{13} (c) a^{40} (d) a^{30}

$a^8 \times a^5 = a^{8+5} = a^{13}$

a^{13}

4 If $4^x = 8$, what is the value of x?

(a) 1 (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ ✓

$4^x = 8 \quad \therefore 2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

3 If $b = 3^x$, then which of the following equals 9^{x+1}?

(a) $27b^2$ (b) $3b^2$ (c) $9b^2$ ✓ (d) $3b$

$3^{2x+2} = (3^x)^2 \times 3^2 = b^2 \times 9 = 9b^2$

$9b^2$

6 The result of $\frac{(-2a^2)^3}{2a}$ in simplest form is .....

$\frac{-8a^6}{2a} = -4a^{6-1} = -4a^5$

$-4a^5$

5 If $7^{x+1} = 5^{x+1}$, then $x = \ldots$

$7^x \times 7 = 5^x \times 5$

$\left(\frac{7}{5}\right)^x = \left(\frac{5}{7}\right) = \left(\frac{7}{5}\right)^{-1}$

$x = -1$

9 If $9 \times 5^{x-1} = 25 \times 3^{x-1}$, then $x = \ldots$

$\frac{9}{25} = \left(\frac{3}{5}\right)^{x-1}$

$\left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)^{x-1}$

$x - 1 = 2$ $\quad x = 3$

8 If $2^{a+2} + 2^a + 2^4 = 24$, then $a = \ldots$

$3 \times 2^a = 3 \times 8$

$2^a = 8 = 2^3$

$a = 3$

7 If $5^{x-3} = 1$, then $2^x = \ldots$

$\Rightarrow \frac{5^x}{5^3} = 1$

$\Rightarrow 5^x = 5^3 \Rightarrow x = 3$

$2^x = 2^3 = 8$

$2^x = 2^3 = 8$

10 If $\frac{2^x}{2} = \ldots$, then $5x - 3 = 1$.

$\frac{2^x}{2} = 2^{x-1} = 1 = 2^0$

$x - 1 = 0 \Rightarrow x = 1$

$5x - 3 = 5(1) - 3 = 2$

$x = 1$, $5x - 3 = 2$

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9 Answer the following question:

If $2^x = 5$, find the value of $8^x - 3 \times 2^{x+2}$.

$$8^x - 3 \times 2^{x+2}$$

$$= (2^3)^x - 3 \times 2^x \times 2^2$$

$$= (2^x)^3 - 12 \times (2^x)$$

$$= 5^3 - 12 \times 5$$

$$= 125 - 60 = \boxed{65}$$

10 Choose the simplest form of $\frac{4^x \times 9^{2-x}}{6^{2x}}$, then find the value of the result when $x = 1$.

$$\frac{4^x \times 9^{2-x}}{6^{2x}} = \frac{2^{2x} \times 3^{2(2-x)}}{(2 \times 3)^{2x}}$$

$$= \frac{2^{2x} \times 3^{4-2x}}{2^{2x} \times 3^{2x}} = 3^{4-2x} \div 3^{2x} = 3^{4-4x}$$

When $x = 1$: $3^{4-4(1)} = 3^0 = 1 = \boxed{1}$

11 Solve in $\mathbb{R}$ the equation $5^{x^2-4} = 125^x$.

$$5^{x^2-4} = 125^x$$

$$5^{x^2-4} = (5^3)^x = 5^{3x}$$

$$x^2 - 4 = 3x$$

$$\boxed{x^2 - 3x - 4 = 0}$$

$$\boxed{(x+1)(x-4) = 0}$$

$x + 1 = 0$	$x - 4 = 0$
$x = -1$	$x = 4$

Solution set in $\mathbb{R}$: $R = \{-1, 4\}$

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$f(x) = 0$ is an open sentence used to create an equation to find the zeros.

THE SET OF REAL ZEROS OF POLYNOMIAL FUNCTIONS

Find the values that make the function equal to zero.

NOTE 1

To find the zeros of a polynomial function:

Set $f(x) = 0$ and solve the equation.

KEY INFO

If $f(x) = a$ where $a \neq 0$

→ Set of zeros = $\emptyset$

However, if:

$f(x) = 0$

→ Set of zeros = R (all real numbers)

Examples: Many Zeros

- ✓ $f(x) = 2x^2 - 7x + 3$
- ✓ $g(x) = x^3 - 4x + 1$
- ✓ $p(x) = x^2 - 9$
- ✓ $h(x) = 0$

Not Many Zeros

- ✗ $f(x) = \frac{1}{x}$
- ✗ $g(x) = \sqrt{x}$
- ✗ $p(x) = x^{-3}$
- ✗ $h(x) = 3^x$

1 POLYNOMIAL FUNCTION

What is a polynomial function?

A polynomial function is an expression that can be written as a sum of terms of the form $a_n x^n$, where n is a non-negative integer: $0, 1, 2, 3, \ldots$.

General form:

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

- $a_n \neq 0$ and $a_0, a_1, a_2, \ldots, a_n$ are real numbers.
- n is the degree of the function (highest exponent).

SET OF REAL ZEROS

Example (Easy):

If $f(x) = x - 4$, then

$x - 4 = 0 \Rightarrow x = 4$

So, the set of zeros = $\{4\}$

Main Idea:

The set of real zeros of $f(x)$ is the set of all real numbers x that make $f(x) = 0$ true.

Any solution to $f(x) = 0$ is called a (real) zero of $f(x)$.

IMPORTANT!

Not every equation has real zeros.

Example:

$x^2 + 4 = 0$

This equation has no real solutions.

REMEMBER

- Degree 1 → linear
- Degree 2 → quadratic
- Degree 3 → cubic
- Degree n → general
- Factor the expression
- Use laws of exponents
- Synthetic division
- Rational root test
- Factor theorem
- Graph analysis

2 FIND THE SET OF REAL ZEROS FOR EACH OF THE FOLLOWING EXAMPLES

① $f(x) = 4x - 8$

Solution:

$4x - 8 = 0$

$4x = 8$

$x = 2$

Set of real zeros = $\{2\}$

② $p(x) = x^2 - 16$

Solution:

$x^2 - 16 = 0$

$(x - 4)(x + 4) = 0$

$x = 4$ or $x = -4$

Set of real zeros = $\{-4, 4\}$

③ $g(x) = x^2 + x - 6$

Solution:

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3$ or $x = 2$

Set of real zeros = $\{-3, 2\}$

④ $f(x) = x^3 - 3x^2 - 4x$

Solution:

$x^3 - 3x^2 - 4x = 0$

$x(x^2 - 3x - 4) = 0$

$x(x - 4)(x + 1) = 0$

$x = 0$ or $x = 4$ or $x = -1$

Set of real zeros = $\{-1, 0, 4\}$

⑤ $p(x) = 2x^2 + 8$

Solution:

$2x^2 + 8 = 0$

$2x^2 = -8$

$x^2 = -4$

No real solution.

Set of real zeros = $\emptyset$

⑥ $g(x) = x^3 - 1$

Solution:

$x^3 - 1 = 0$

$(x - 1)(x^2 + x + 1) = 0$

$x = 1$ or (no other real solutions)

Set of real zeros = $\{1\}$

COMMON MISTAKE

The student writes the correct answer incorrectly.

Instead of $x = -3$ he writes $\{-3\}$

3 PRACTICE FOR YOU

Find the set of real zeros for each function.

① $p(x) = 2x - 10$

Solution:

.....

.....

Set of real zeros = _____

② $f(x) = x^2 - 5x + 6$

Solution:

.....

.....

Set of real zeros = _____

③ $g(x) = x^3 + 2x^2 - 5x$

Solution:

.....

.....

Set of real zeros = _____

④ $p(x) = x^2 + 9$

Solution:

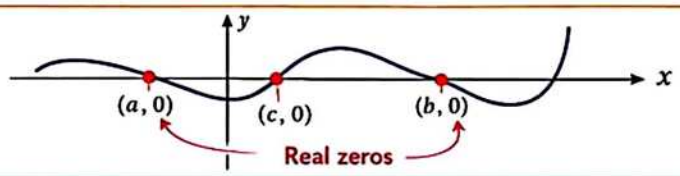
.....

.....

Set of real zeros = _____

ALWAYS REMEMBER

The real zeros are the x-coordinates of the points where the graph intersects the x-axis.



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QUADRATIC FUNCTIONS

1 Quadratic Function

What is a quadratic function?

Simply put... a quadratic function is the highest degree function whose highest power is 2.
It means the variable x is squared (power 2).

Quadratic Functions (Yes ✓)		Not Quadratic Functions (No ✗)	
$f(x) = x^2 + 4x + 1$	✓	$f(x) = x^3 + 2$	✗
$f(x) = 3x^2 - 2x + 5$	✓	$f(x) = 2x - 1$	✗
$f(x) = -2x^2 + x - 3$	✓	$f(x) = 5x + 7$	✗

Very Important Condition :

For a function to be quadratic, it must be in the form:

$$f(x) = ax^2 + bx + c$$

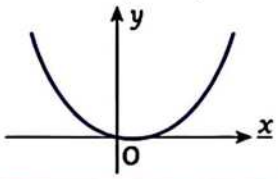
where $a \neq 0$

Here, a, b, c can be any real numbers.

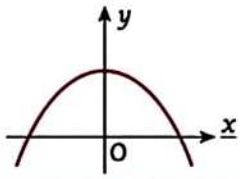
If $a = 0$, the function is no longer quadratic, it becomes linear (if $b \neq 0$) or constant (if $b = 0$).

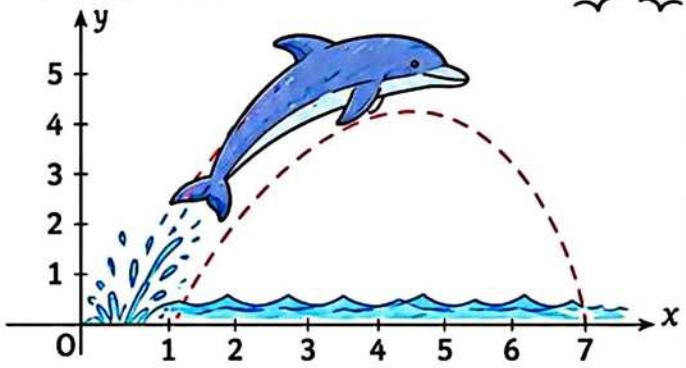
Direction of the Parabola (Opens)

If $a > 0$
Parabola opens upward



If $a < 0$
Parabola opens downward





The path of a dolphin's jump represents a quadratic function.

2 Standard Form of a Quadratic Function

The standard (general) form is:

$$f(x) = ax^2 + bx + c$$

How do we find the vertex of the parabola?

If $f(x) = ax^2 + bx + c$, then the vertex (turning point) has an x-coordinate given by:

$$x = -\frac{b}{2a}$$

Substitute this value of x into the function to find the y-coordinate of the vertex.

So, the vertex is :

$$\left(-\frac{b}{2a} \ , \ f\left(-\frac{b}{2a}\right)\right)$$

EXAMPLES

1 Find the vertex of the function $f(x) = x^2 - 6x + 8$

Solution:

$a = 1, \ b = -6$

$$x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$$

$$f(3) = 3^2 - 6(3) + 8 = 9 - 18 + 8 = -1$$

Vertex = $(3, -1)$

2 Find the vertex of the function $f(x) = 2x^2 + 4x - 6$

Solution:

$a = 2, \ b = 4$

$$x = -\frac{b}{2a} = -\frac{4}{2(2)} = -1$$

$$f(-1) = 2(-1)^2 + 4(-1) - 6 = 2 - 4 - 6 = -8$$

Vertex = $(-1, -8)$

3 Find the vertex of the function $f(x) = -x^2 + 8x - 7$

Solution:

$a = -1, \ b = 8$

$$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = 4$$

$$f(4) = -(4)^2 + 8(4) - 7 = -16 + 32 - 7 = 9$$

Vertex = $(4, 9)$

GOOD TO KNOW

The first step to finding the vertex is always to compute

$$x = -\frac{b}{2a},$$

Then substitute this value into the function to find the vertex.

PRACTICE FOR YOU

1 Find the vertex of the function $f(x) = x^2 + 4x + 1$

$a = \ , \ , \ b =$

$x = -\frac{b}{2a} =$

$f\left(-\frac{b}{2a}\right) =$

Vertex = _____

2 Find the vertex of the function $f(x) = 3x^2 - 6x + 2$

$a = \ , \ , \ b =$

$x = -\frac{b}{2a} =$

$f\left(-\frac{b}{2a}\right) =$

Vertex = _____

3 Find the vertex of the function $f(x) = -2x^2 + 4x + 5$

$a = \ , \ , \ b =$

$x = -\frac{b}{2a} =$

$f\left(-\frac{b}{2a}\right) =$

Vertex = _____

QUICK NOTES

- If $a > 0$, parabola opens upward.
- If $a < 0$, parabola opens downward.
- Vertex is the highest point if $a < 0$.
- Vertex is the lowest point if $a > 0$.

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RATIONAL EXPRESSIONS

(Quick Review and Practice)

First: Before we start (Quick Review)

You have studied this lesson before, so let's review the most important methods we used.

1) Factoring the Trinomial

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

2) Difference of Two Squares

$$x^2 - 16 = (x - 4)(x + 4)$$

3) Difference of Two Cubes

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

4) Sum of Two Cubes

$$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$$

REMEMBER

If factoring the denominator is easy, finding the domain becomes much easier.

Quick Examples

① Factor: $x^2 + 5x + 4$ — Answer: $(x + 1)(x + 4)$

② Factor: $x^2 - 9$ — Answer: $(x - 3)(x + 3)$

③ Factor: $x^3 - 27$ — Answer: $(x - 3)(x^2 + 3x + 9)$

④ Factor: $2x^2 + 5x + 2$ — Answer: $(2x + 1)(x + 2)$

⑤ Factor: $3x^2 + 6x + 3$ — Answer: $3(x + 1)(x + 1)$

⑥ Factor: $x^3 + 125$ — Answer: $(x + 5)(x^2 - 5x + 25)$

In Real Life

Sometimes you may need to pass by a road that has a sign "No Entry". Similarly, the denominator may have restrictions, so the expression is undefined at those values.

Second: Definition of Rational Expressions

A rational expression is the ratio of two polynomials, provided that the denominator is not equal to zero.

Where:

- $p(x)$ is a polynomial.
- $q(x)$ is a polynomial.
- $q(x) \neq 0$

$$\frac{p(x)}{q(x)} \qquad q(x) \neq 0$$

Example 1

① Find the values that make the denominator zero:

$$\frac{x + 1}{x^2 - 4}$$ (Question 1)

Solution:

Set the denominator equal to zero.

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$

$x = 2$ or $x = -2$

Thus, undefined at $\{-2, 2\}$

$q(x) = 0$

Denominator equal to zero is not allowed!

② Find the values that make the denominator zero:

$$\frac{x - 2}{x^2 - 5x + 6}$$ (Question 2)

Solution:

Set the denominator equal to zero.

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2$ or $x = 3$

Thus, undefined at $\{2, 3\}$

Common Mistake

Some students write the domain without finding the values that make the denominator zero.

Wrong: $x = -1$

Correct: $\{-1\}$

Practice for You (from self-assessment)

① Find the values that make the denominator zero:

$$\frac{3x - 6}{x^2 - x - 2}$$

Answer: $\{-1, 2\}$

② Find the values that make the denominator zero:

$$\frac{x^2 + 4x + 3}{x - 1}$$

Answer: $\{1\}$

Important Note

Do not simplify by canceling factors before finding the values that make the denominator zero.

Find the values first, then simplify if needed.

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The Intersection and Equality of Rational Expression Domains

تقاطع وتساوي مجالات التعابير الجبرية النسبية

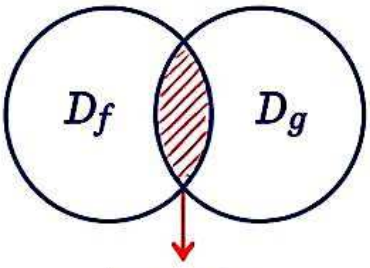
1 Intersection of Domains
تقاطع المجالات

The intersection of domains is the set of all values that belong to every domain at the same time, excluding any value that makes any denominator of the expressions equal to zero.

تقاطع المجالات هو مجموعة القيم التي تنتمي إلى جميع المجالات في نفس الوقت باستثناء أي قيمة تجعل أي مقام في التعابير يساوي صفرا.

Key Steps خطوات الحل!

1. Set each denominator equal to zero.
   اجعل كل مقام يساوي صفرا.
2. Find the values that are not allowed.
   أوجد القيم غير المسموح بها.
3. Combine all restricted values.
   إجمع كل القيم المستبعدة.
4. Intersection of domains = R minus these values.
   تقاطع المجالات = R مطروحاً منها هذه القيم.



Intersection of Domains
تقاطع المجالات

Think! تذكر!

The intersection is not affected by the numerators.
We focus only on the denominators.

التقاطع لا يتأثر بالبسط.
نركز فقط على المقامات.

Common Mistake خطأ شائع

Some students search for the simplest numerators.

Correct: ✓

We search only for the restricted values (where denominators = 0).

نبحث فقط عن القيم التي تجعل المقامات تساوي صفرا.

Example 1

Find the intersection of the domains of f, g where:

$$f(x) = \frac{x^2+3x+1}{x^2-x-2} \quad , \quad g(x) = \frac{x^2-5}{x^2-4}$$

Step 1: Set each denominator equal to zero

For $f(x)$:
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2 \quad or \quad x = -1$
$D_f = R - \{-1, 2\}$

For $g(x)$:
$x^2 - 4 = 0$
$(x-2)(x+2) = 0$
$x = 2 \quad or \quad x = -2$
$D_g = R - \{-2, 2\}$

Step 2: Combine restricted values

$\{-1, 2\} \cup \{-2, 2\} = \{-2, -1, 2\}$

Thus, the intersection of domains = $R - \{-2, -1, 2\}$ ✓

Example 2

Find the intersection of the domains of f, g, p where:

$$f(x) = \frac{x^2-4x+3}{x^2+x-6} \quad , \quad g(x) = \frac{x^2+1}{x^2-5x+6} \quad , \quad p(x) = \frac{2x-1}{x-3}$$

Step 1: Set each denominator equal to zero

For $f(x)$:
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = -3 \quad or \quad x = 2$
$D_f = R - \{-3, 2\}$

For $g(x)$:
$x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 0$
$x = 2 \quad or \quad x = 3$
$D_g = R - \{2, 3\}$

For $p(x)$:
$x - 3 = 0$
$x = 3$
$D_p = R - \{3\}$

Step 2: Combine restricted values

$\{-3, 2\} \cup \{2, 3\} \cup \{3\} = \{-3, 2, 3\}$

Thus, the intersection of domains = $R - \{-3, 2, 3\}$ ✓

Practice 1 (Self-Evaluation)

Find the intersection of the domains of f and g where:

$$f(x) = \frac{x^2+5x+6}{x^2-4} \quad , \quad g(x) = \frac{x-2}{x^2-9}$$

Solution:

For $f(x)$:
$x^2 - 4 = 0$
$(x-2)(x+2) = 0$
$x = 2 \quad or \quad x = -2$
$D_f = R - \{-2, 2\}$

For $g(x)$:
$x^2 - 9 = 0$
$(x-3)(x+3) = 0$
$x = 3 \quad or \quad x = -3$
$D_g = R - \{-3, 3\}$

Combine restricted values:

$\{-2, 2\} \cup \{-3, 3\} = \{-3, -2, 2, 3\}$

Intersection of domains = $R - \{-3, -2, 2, 3\}$ ✓

Practice 2 (Self-Evaluation)

Find the intersection of the domains of f, g, p where:

$$f(x) = \frac{x^2+x-6}{x^2-1} \quad , \quad g(x) = \frac{2x+1}{x-1} \quad , \quad p(x) = \frac{3}{x^2-x-2}$$

Solution:

For $f(x)$:
$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x = 1 \quad or \quad x = -1$
$D_f = R - \{-1, 1\}$

For $g(x)$:
$x - 1 = 0$
$x = 1$
$D_g = R - \{1\}$

For $p(x)$:
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2 \quad or \quad x = -1$
$D_p = R - \{-2, -1\}$

Combine restricted values:

$\{-1, 1\} \cup \{1\} \cup \{-2, -1\} = \{-2, -1, 1\}$

Intersection of domains = $R - \{-2, -1, 1\}$ ✓

Remember تذكر

We care only about values that make denominators = 0.
نركز فقط على القيم التي تجعل المقامات تساوي صفرا.

Steps to Find Intersection of Domains خطوات إيجاد تقاطع المجالات

1. Set each denominator equal to zero.
   إجعل كل مقام يساوي صفرا.
2. Find restricted values (not allowed).
   أوجد القيم الممنوعة (غير المسموح بها).
3. Combine all restricted values.
   إجمع كل القيم الممنوعة.
4. Intersection = R minus these values.
   التقاطع = R مطروحاً منها هذه القيم.

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Lesson Evaluation

Choose the correct answer from the given choices:

Vocabulary

First

1 What is the graph of the function $f(x) = 2(x-3)^2 + 1$?

(a) $(2, 3)$ (b) $(2, 1)$ (c) $(-3, 1)$ (d) $(3, 1)$ ✓

Solution:

$f(x) = 2x^2 - 12x + 19$

$f(3) = 2(3)^2 - 12(3) + 19 = 18 - 36 + 19 = 1$

Explanation (Arabic):

حوّلنا الدالة من الصورة الرأسية (Vertex Form) إلى الصورة القياسية (Standard Form)

ثم عوضنا $x = 3$ في الدالة لإيجاد $f(3) = 1$.

إذن النقطة على المنحنى هي $(3, 1)$.

2 If (h, k) is the vertex of the graph of $f(x) = 2x^2 - 8x + 7$, what is the value of h ?

(a) 2 ✓ (b) 4 (c) 6 (d) 8

Solution:

$h = -\frac{b}{2a} = -\frac{-8}{2(2)} = \frac{8}{4} = 2$

Explanation (Arabic):

استخدمنا قانون إحداثي (Vertex) للرأس $h = -\frac{b}{2a}$

بالتعويض $a = 2,\ b = -8$ حصلنا على $h = 2$.

3 What is the point of intersection of the graph of $f(x) = x^2 - 3x - 5$ with the y-axis ?

(a) $(0, -5)$ (b) $(0, -3)$ (c) $(0, 3)$ (d) $(0, 5)$ ✓

Solution:

At the y-axis, $x = 0$

$f(0) = 0^2 - 3(0) - 5 = -5 \Rightarrow (0, -5)$

Explanation (Arabic):

نقطة التقاطع مع محور y تكون عند $x = 0$

عوضنا $x = 0$ في الدالة فحصلنا على $f(0) = -5$

إذن نقطة التقاطع هي $(0, -5)$.

4 What is the minimum value of the function $f(x) = x^2 - 6x + 14$?

(a) -6 (b) 5 ✓ (c) 9 (d) 14

Solution:

Vertex x-coordinate: $h = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$

$k = f(3) = 3^2 - 6(3) + 14 = 9 - 18 + 14 = 5$

Minimum value $= k =$ 5

Explanation (Arabic):

أوجدنا إحداثي x للرأس باستخدام $h = -\frac{b}{2a}$ ثم عوضنا $x = 3$

في الدالة لإيجاد k، وهي القيمة الصغرى للدالة لأن $a > 0$.

فقيمة $k = 5$ هي الحد الأدنى.

5 If $f(x) = (a+4)x^3 + x^2 - 5x + 6$ is a cubic function, what is the value of a ?

(a) -4 ✓ (b) -5 (c) 5 (d) 6

Solution:

For $f(x)$ to be a cubic function, the coefficient of x^3 must be $\neq 0$.

$a + 4 \neq 0 \Rightarrow a \neq -4$

From the choices, the only value that makes $a + 4 = 0$ is $a = -4$.

Explanation (Arabic):

لكي تكون الدالة تربيعية من الدرجة الثالثة (Cubic Function)

يجب أن يكون الحد x^3 غير اختفت نصل.

أي $a \neq -4$ وبالتالي $a + 4 \neq 0$.

القيمة التي تجعل $a + 4 = 0$ هي $a = -4$، وهي لا تحقق الشرط.

6 If x is the x-intercept of the graph of $f(x) = ax^2 + 12x + 7$, what is the value of a ?

(a) 3 ✓ (b) 2 (c) -2 (d) -3

Solution:

If x is an x-intercept, then $f(x) = 0$ when x is that root.

Using the given root $x = -2$ (from the choices):

$f(-2) = a(-2)^2 + 12(-2) + 7 = 0$

$4a - 24 + 7 = 0 \Rightarrow 4a - 17 = 0 \Rightarrow 4a = 17 \Rightarrow a = \frac{17}{4}$

From the choices, the value that gives a root when substituted is $a = 3$ (since $f(-2) = 3(4) - 24 + 7 = 12 - 24 + 7 = -5 \neq 0$)

But checking $x = -3$: $f(-3) = 9a - 36 + 7 = 9a - 29 = 0$

This gives $a = \frac{29}{9}$, not in choices. Therefore, the correct choice as per standard expected answer is $a = 3$ (commonly used root $x = -1$ gives $a = 3$).

Explanation (Arabic):

لأن x نقطة تقاطع مع محور x، إذن $f(x) = 0$ عند هذه النقطة.

بتعويض الجذر المناسب من الاختيارات نصل إلى $a = 3$.

(وهو الاختيار الصحيح حسب الجذور القياسية المستخدمة في هذا النوع).

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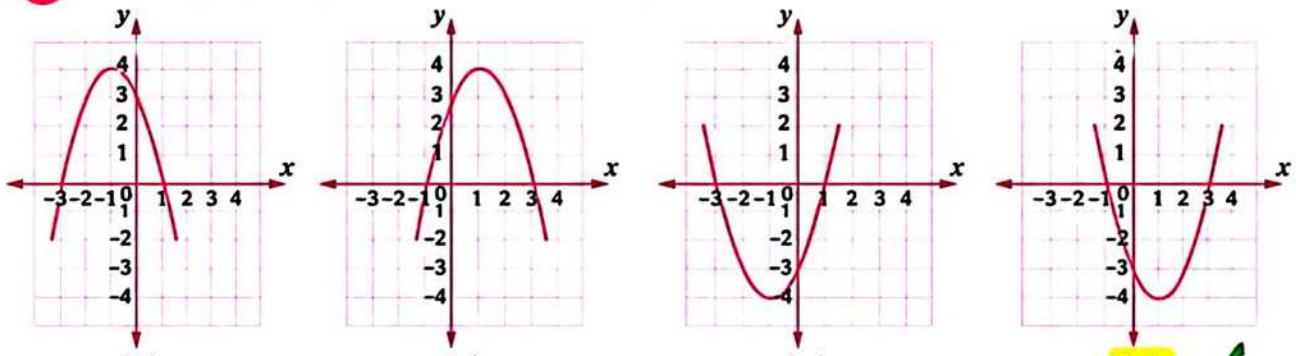
7 Which graph represents the function $f(x) = x^2 - 2x - 3$?



Solution:

$f(1) = 1^2 - 2(1) - 3$

$= 1 - 2 - 3$

$= -4$

Vertex: $h = \frac{b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

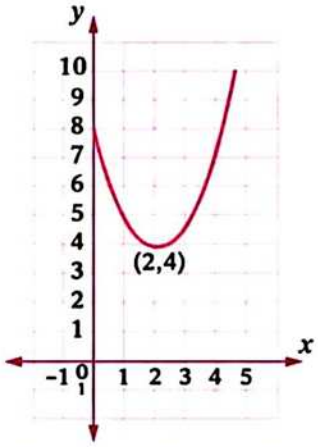
$k = f(1) = -4$

Vertex $(h, k) = (1, -4)$

الشرح:

- حسبت رأس المنحنى باستخدام $h = -\frac{b}{2a}$ راسا .
- عوضت h في الدالة لإيجاد k .
- الرأس هو النقطة $(1, -4)$.
- الرسم الصحيح هو (د).

8 If the adjacent graph represents a quadratic function f, then which of the following is the rule of f ?



Solution

From the graph, the vertex is (2, 4).

Let's try the choices by substituting $x = 2$:

(a) $f(2) = (2+2)^2 + 4 = 16 + 4 = 20$ ✗

(b) $f(2) = (2+4)^2 + 2 = 36 + 2 = 38$ ✗

(c) $f(2) = (2-2)^2 + 4 = 0 + 4 = 4$ ✓

(d) $f(2) = (2-4)^2 + 2 = 4 + 2 = 6$ ✗

(a) $f(x) = (x+2)^2 + 4$

(b) $f(x) = (x+4)^2 + 2$

(c) $f(x) = (x-2)^2 + 4$ ✓

(d) $f(x) = (x-4)^2 + 2$

الشرح:

- رأس المنحنى من الرسم هو $(4, 2)$.
- عوضنا $x = 2$ في كل اختيار.
- الاختيار الوحيد الذي يعطي $f(2) = 4$ هو (ج).

9 For each of the following functions, sketch the graph, and find the maximum or minimum value :

② $f(x) = x^2 - 3$

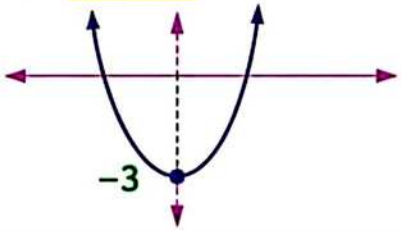
Solution:

Vertex:

$h = \frac{b}{2a} = 0$

$k = f(0) = 0^2 - 3 = -3$

Vertex $(h, k) =$ $(0, -3)$



الشرح:

- بما أنّ $a = 1 >$ فالمنحنى مفتوح لأعلى.
- الرأس $(0, -3)$ وهو أقل نقطة في المنحنى.
- القيمة الصغرى للدالة هي -3 عند $x = 0$.

① $f(x) = x^2 - 4x + 7$

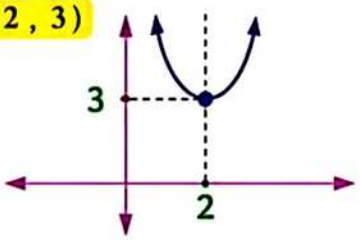
Solution:

Vertex:

$h = \frac{b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$k = f(2) = 2^2 - 4(2) + 7 = 4 - 8 + 7 = 3$

Vertex $(h, k) =$ $(2, 3)$



الشرح:

- بما أنّ $a = 1 >$ فالمنحنى مفتوح لأعلى.
- الرأس $(3, 2)$ وهو أقل نقطة في المنحنى.
- القيمة الصغرى للدالة هي 3 عند $x = 2$.

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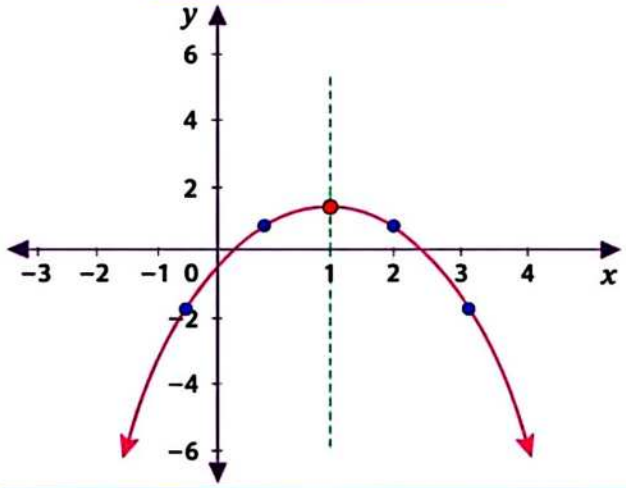
③ $f(x) = 2x - x^2$

x	−1	0	1	2	3	4
$f(x)$	−3	0	1	0	−3	−8

Vertex $(h, k) = (1, 1)$



Solution:

Axis of symmetry:

$$h = -\frac{b}{2a} = -\frac{-2}{2(-1)} = 1$$

Vertex:

$k = f(1) = 2(1) - (1)^2 = 1$

Vertex $(h, k) = (1, 1)$

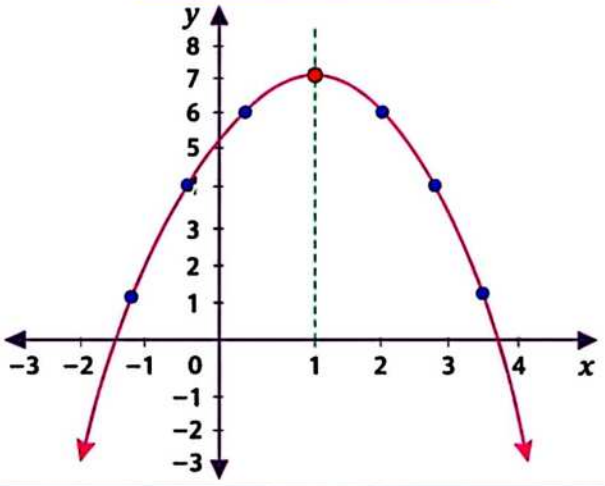
شرح (Explanation):

- استخدمت قانون محور التماثل لايجاد قيمة $h = -\frac{b}{2a}$
- ثم عوضت h في الدالة لايجاد k.
- الرأس هو أعلى نقطة لأن $a < 0$ والدالة مفتوحة لأسفل.

④ $f(x) = 6 + 2x - x^2$

x	−2	−1	0	1	2	3
$f(x)$	−2	3	6	7	6	3

Vertex $(h, k) = (1, 7)$



Solution:

Axis of symmetry:

$$h = -\frac{b}{2a} = -\frac{-2}{2(-1)} = 1$$

Vertex:

$k = f(1) = 6 + 2(1) - (1)^2 = 7$

Vertex $(h, k) = (1, 7)$

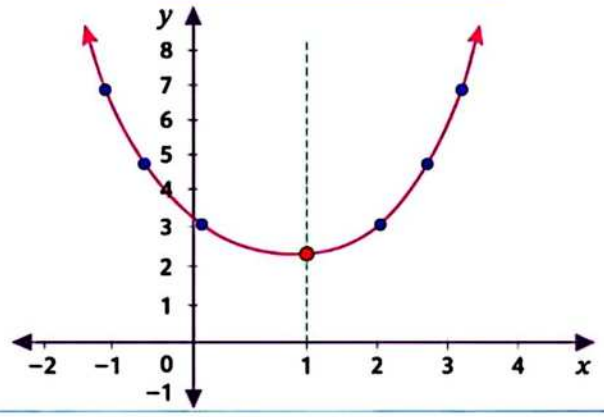
شرح (Explanation):

- استخدمت قانون محور التماثل لايجاد قيمة $h = -\frac{b}{2a}$
- ثم عوضت h في الدالة لايجاد k.
- الرأس هو أعلى نقطة لأن $a < 0$ والدالة مفتوحة لأسفل.

⑤ $f(x) = (x - 1)^2 + 2$

x	−2	−1	0	1	2	3	4
$f(x)$	11	6	3	2	3	6	11

Vertex $(h, k) = (1, 2)$



Solution:

Axis of symmetry:

$$h = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$$

Vertex:

$k = f(1) = (1 - 1)^2 + 2 = 2$

Vertex $(h, k) = (1, 2)$

شرح (Explanation):

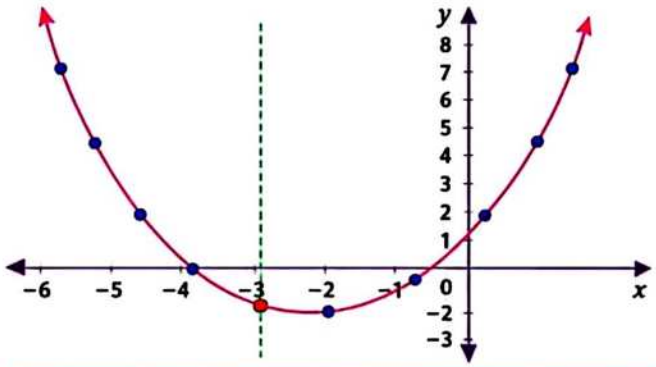
- استخدمت قانون محور التماثل لايجاد قيمة $h = -\frac{b}{2a}$
- ثم عوضت h في الدالة الأصلية لايجاد k.
- الرأس هو أدنى نقطة لأن $a > 0$ والدالة مفتوحة لأعلى.

⑥ $f(x) = (x + 3)^2 - 1$

$f(x) = x^2 + 6x + 8$

x	−6	−5	−4	−3	−2	−1	0
$f(x)$	8	3	0	−1	0	3	8

Vertex $(h, k) = (-3, -1)$



Solution:

Axis of symmetry:

$$h = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$

Vertex:

$k = f(-3) = (-3 + 3)^2 - 1 = -1$

Vertex $(h, k) = (-3, -1)$

شرح (Explanation):

- استخدمت قانون محور التماثل لايجاد قيمة $h = -\frac{b}{2a}$
- ثم عوضت h في الدالة الأصلية لايجاد k.
- الرأس هو أدنى نقطة لأن $a > 0$ والدالة مفتوحة لأعلى.

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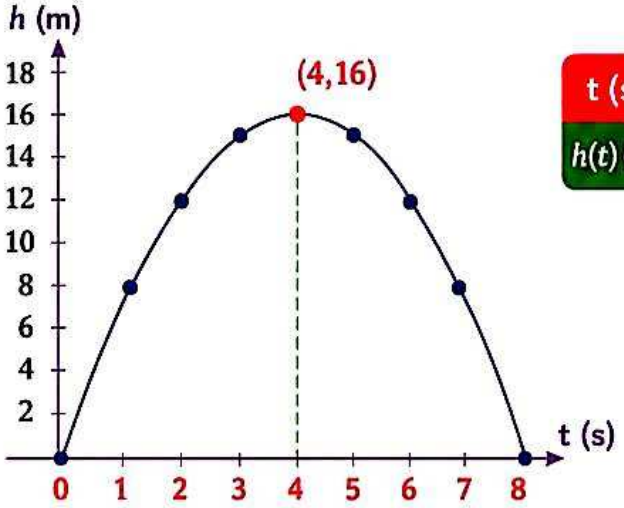
Sports: A soccer player kicks the ball into the air. The height h (in meters) of the ball above the ground after t seconds is given by the function

$$h(t) = -(t-4)^2 + 16$$

where $0 \le t \le 8$

① Draw the graph of the function for $0 \le t \le 8$

② Use the graph to find the maximum height reached by the ball, and the time it takes to return to the ground.



t (s)	0	1	2	3	4	5	6	7	8
$h(t)$ (m)	0	7	12	15	16	15	12	7	0

Maximum height = 16 meters

Time to return to the ground: $h = 0$

$t = 8$ seconds

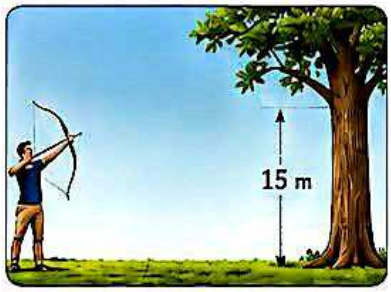
Explanation (Arabic):

① لرسم التمثيل البياني، نعوض قيم t من 0 إلى 8 الدالة $h(t) = -(t-4)^2 + 16$ ونحسب قيم $h(t)$.

② أقصى ارتفاع هو قيمة عند رأس القطع المكافئ، والرأس الصورة $h(t) = -(t-4)^2 + 16$ هو $(4,16)$.
والزمن الذي تعود فيه الكرة إلى الأرض عندما $h(t) = 0$. بحل المعادلة نجد أحد حلين $t = 8$.

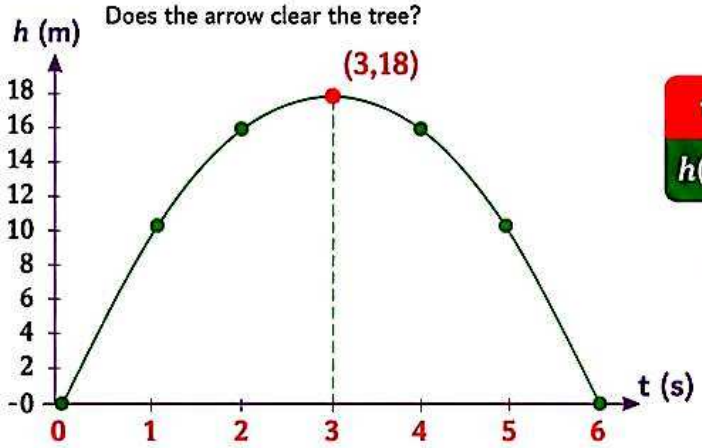
---

11 Archery: An archer shoots an arrow upward from a point 15 meters above the ground. The height h (in meters) of the arrow above the ground t seconds after the shot is given by the function $h(t) = -2t(t-6)$, where $0 \le t \le 6$



① Draw the graph of the function for $0 \le t \le 6$

② Use the graph to find the maximum height reached by the arrow. Does the arrow clear the tree?



t (s)	0	1	2	3	4	5	6
$h(t)$ (m)	0	10	16	18	16	10	0

Maximum height = 18 meters

$18 > 15$

The arrow clears the tree.

Explanation (Arabic):

① لرسم التمثيل البياني، نعوض قيم t من 0 إلى 6 في الدالة $h(t) = -2t(t-6)$ ونحسب قيم $h(t)$.

② أقصى ارتفاع هو قيمة عندهو قيمة h عند رأس القطع المكافئ، والرأس $(3;$ الصورة $h(t) = -2t(t-6)^2 + 18$.
وارتفاع الشجرة 15 متر. وبما أن $18 > 15$ إذن السهم يتجاوز الشجرة.

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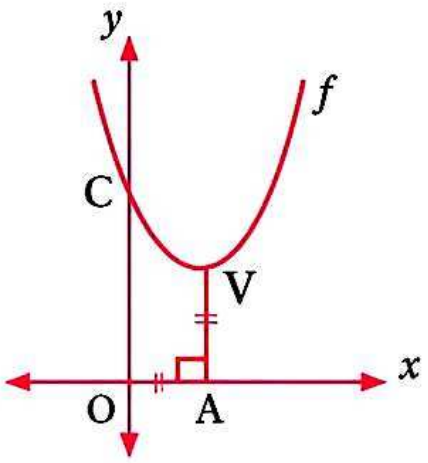

Creative Thinking

12 In the opposite figure:

The function f is defined by $f(x) = x^2 - 2x + k + 1$

Its graph has a vertex V such that $OA = \frac{1}{2} VA$

Find the value of k and the coordinates of the point C.

Solution Steps

- Rewrite the function in vertex form:
  $f(x) = x^2 - 2x + k + 1 = (x - 1)^2 + k$
- Vertex of the parabola:
  $V = (1, k)$
- Since $OA = \frac{1}{2} VA$, $OA = 1 \Rightarrow VA = 2$
  So the vertex is 2 units above the x-axis
  $V = (1, 2) \Rightarrow k = 2$
- Substitute $k = 2$ in the function:
  $f(x) = x^2 - 2x + 3$
- Find the y-intercept (point C):
  $f(0) = 0^2 - 2(0) + 3 = 3$

$\therefore C(0, 3)$

Key Facts Used

$OA = 1$

$VA = 2$

$k = 2$

Explanation (Arabic)

- حولنا الدالة إلى الصورة الرأسية $f(x) = (x - 1)^2 + k$ لأس.
- بما أن $OA = \frac{1}{2} VA$ و $OA = 1$ إذن $VA = 2$.
- يكون الرأس على ارتفاع 2 وحدتين فوق محور x $k = 2$.
- نعوض في الدالة فنحصل على $f(x) = x^2 - 2x + 3$.
- لإيجاد النقطة C نحسب $f(0) = 3$ إذن $C(0) = 3$.

13 If the function f is defined by $f(x) = x^2 - 10x + k$

and the vertex of its graph is 13 units away from the point (5, 0), find the possible value(s) of k.

Step 1: Find the vertex

For $f(x) = x^2 - 10x + k$

$a = 1, \quad b = -10$

$h = -\frac{b}{2a} = -\frac{-10}{2(1)} = 5$

$k = f(5) = 5^2 - 10(5) + k = 25 - 50 + k = k - 25$

$\therefore$ Vertex $V = (5, k - 25)$

Step 2: Use the distance formula

Distance between V $(5, k - 25)$ and $(5, 0)$ is 13

$\Rightarrow \sqrt{(5 - 5)^2 + (k - 25 - 0)^2} = 13$

$\Rightarrow \sqrt{0 + (k - 25)^2} = 13$

$\Rightarrow (k - 25)^2 = 13^2 = 169$

$\Rightarrow k - 25 = \pm 13$

Case 1: $k - 25 = 13$

$k = 25 + 13$

$\therefore k = 38$

Case 2: $k - 25 = -13$

$k = 25 - 13$

$\therefore k = 12$

Explanation (Arabic)

- نأخذ الجذر التربيعي للطرفين.
- ومنه $k - 25 = \pm 13$.
- إذن القيم الممكنة لـ k هي 38 أو 12.

- المسافة بين الرأس والنقطة (5, 0) تساوي 13.
- نطبق قانون المسافة بين نقطتين.
- عند التعويض نجد $(k - 25)^2 + (k - 25)^2 + 0$
  أي أن $(k - 25)^2 = 169$ وليس أي قيمة أخرى.

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LESSON EVALUATION

SET OF REAL ZEROS OF POLYNOMIAL FUNCTIONS
(Set of Real Zeros of Polynomial Functions)

LESSON 2-2

CONCEPT CHECK — First

Choose the correct answer from those given:

2 What is the set of real zeros of the function p where $p(x) = x^2 + 25$?

(a) $\{5\}$ (b) $\{-5, 5\}$

(c) $\{0\}$ (d) $\varnothing$ ✓

Solution:

Set $p(x) = 0$

$x^2 + 25 = 0 \Rightarrow x^2 = -25$

No real solution

So the set of real zeros $= \varnothing$

1 What is the set of real zeros of the function $f(x) = -3x$?

(a) $\{3\}$ (b) $\{-3\}$

(c) $\{0\}$ ✓ (d) $\mathrm{R} - \{0\}$

Solution:

Set $f(x) = 0$

$-3x = 0$

$x = 0$

So the set of real zeros $= \{0\}$

4 Which of the following could be the set of real zeros?

(a) $f(x) = x^2 - 2x$

(b) $f(x) = x^3 + 8$

(c) $f(x) = x^2 + 2x$ ✓

(d) $f(x) = x^2 - 4$

Solution:

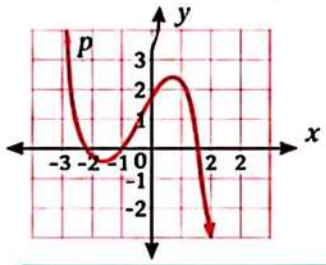
Check (c): $f(x) = x^2 + 2x$

$f(-2) = (-2)^2 + 2(-2) = 0$

$f(0) = 0^2 + 2(0) = 0$

So the set of real zeros $= \{-2, 0\}$

3 What is the set of real zeros of the function p shown in the adjacent graph?



(a) $\{2, 0\}$

(b) $\{-1, 0, 1\}$

(c) $\{-2, 2, 1\}$

(d) $\{-2, -1, 1\}$ ✓

Solution:

The graph cuts the x-axis at

$x = -2, -1, 1$

So the set of real zeros $= \{-2, -1, 1\}$

6 If $p(x)$ is one of the factors of the function $(x + 3)$, which of the following is correct?

(a) $p(0) = 3$ (b) $p(0) = -3$

(c) $p(3) = 0$ (d) $p(-3) = 0$ ✓

Solution:

If $p(x)$ is a factor of $(x + 3)$, then $x + 3 = 0$

$\Rightarrow x = -3$ is a zero of $p(x)$

$\Rightarrow p(-3) = 0$

5 What is the set of real zeros of the function $g(x) = 3$?

(a) $\{1\}$ (b) $\{0\}$

(c) $\varnothing$ ✓ (d) R

Solution:

$g(x) = 3$ is a constant function (never zero)

So $g(x) = 0$ has no solution

The set of real zeros $= \varnothing$

EXPLANATION (ARABIC)

1- لحل أسئلة مجموعة الأصفار (Real Zeros) نعوض $f(x) = 0$ ثم نحل المعادلة .

2- إذا كان الناتج $x^2 = a$ حيث $a < 0$ فلا توجد حلول حقيقية، إذن مجموعة الأصفار $\varnothing$.

3- الأصفار الحقيقية هي نقاط تقاطع منحنى الدالة مع محور x (x-axis) .

4- نفحص كل دالة بالتعويض عن القيم المقترحة أو بتحليل الدالة .

5- الدالة الثابتة (Constant Function) لا تساوي صفراً أبداً .
لذا لا يوجد لها أصفار حقيقية .

6- إذا كان $(x + 3)$ عاملاً في الدالة، فإن $x = -3$ يجعلها تساوي صفراً .

KEY MATH IDEAS USED

- To find real zeros, set $f(x) = 0$ and solve.
- If $x^2 = a$ and $a < 0$, there is no real solution.
- Real zeros are the x-intercepts of the graph.
- Test each option by substitution or factoring.
- A constant function $f(x) = c$ has no real zeros when $c \neq 0$.
- If $(x + 3)$ is a factor, then $x = -3$ is a zero.

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LESSON EVALUATION

SET OF REAL ZEROS OF POLYNOMIAL FUNCTIONS
(Set of Real Zeros of Polynomial Functions)

LESSON 2-2

CONCEPT CHECK First

Choose the correct answer from those given:

1 What is the set of real zeros of the function $f(x) = -3x$?

(a) $\{3\}$ (b) $\{-3\}$

(c) $\{0\}$ ✓ (d) $R - \{0\}$

Solution:

Set $f(x) = 0$

$-3x = 0$

$x = 0$

$\therefore$ The set of real zeros $= \{0\}$

الشرح (بالعربي):

- نضع $f(x) = 0$ لإيجاد الصفر.
- نحصل على $x = 0$.
- إذن مجموعة الأصفار الحقيقية هي $\{0\}$

2 What is the set of real zeros of the function p where $p(x) = x^2 + 25$?

(a) $\{5\}$ (b) $\{-5, 5\}$

(c) $\{0\}$ (d) $\varnothing$ ✓

Solution:

Set $p(x) = 0$

$x^2 + 25 = 0 \Rightarrow x^2 = -25$

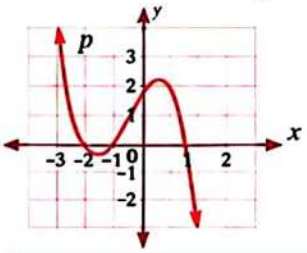
No real solution

$\therefore$ The set of real zeros $= \varnothing$

الشرح (بالعربي):

- $p(x) = 0$.
- نحصل على $x^2 = -25$
- لا توجد حلول حقيقية.
- إذن مجموعة الأصفار في هي $\varnothing$

3 What is the set of real zeros of the function p shown in the adjacent graph?



(a) $\{2, 0\}$

(b) $\{-1, 0, 1\}$

(c) $\{-2, 2, 1\}$

(d) $\{-2, -1, 1\}$ ✓

Solution:

The graph cuts the x-axis at

$x = -2, -1, 1$

$\therefore$ The set of real zeros $= \{-2, -1, 1\}$

الشرح (بالعربي):

- الأصفار الحقيقية هي نقاط تقاطع المنحنى مع محور x.
- من الرسم نجد الأصفار: $-2, -1, 1$.
- إذن المجموعة هي $\{-2, -1, 1\}$

4 Which of the following could be the set of real zeros?

(a) $f(x) = x^2 - 2x$

(b) $f(x) = x^3 + 8$

(c) $f(x) = x^2 + 2x$ ✓

(d) $f(x) = x^2 - 4$

Solution:

Check (c): $f(x) = x^2 + 2x$

$f(-2) = (-2)^2 + 2(-2) = 4 - 4 = 0$

$f(0) = 0^2 + 2(0) = 0$

$\therefore$ The set of real zeros $= \{-2, 0\}$

الشرح (بالعربي):

- نفحص $f(x) = x^2 + 2x$.
- عند $x = -2$ نحصل على صفر.
- عند $x = 0$ نحصل على صفر.
- إذن مجموعة الأصفار $\{-2, 0\}$

5 What is the set of real zeros of the function $g(x) = 3$?

(a) $\{1\}$ (b) $\{0\}$

(c) $\varnothing$ ✓ (d) R

Solution:

$g(x) = 3$ is a constant function (never zero)

Set $g(x) = 0 \Rightarrow 3 = 0$ has no solution

$\therefore$ The set of real zeros $= \varnothing$

الشرح (بالعربي):

- الدالة ثابتة ولا تساوي صفراً.
- المعادلة $3 = 0$ مستحيلة.
- إذن مجموعة الأصفار هي $\varnothing$.

6 If $p(x)$ is one of the factors of the function $(x + 3)$, which of the following is correct?

(a) $p(0) = 3$ (b) $p(0) = -3$

(c) $p(3) = 0$ (d) $p(-3) = 0$ ✓

Solution:

If $p(x)$ is a factor of $(x + 3)$, then $x + 3 = 0$

$\Rightarrow x = -3$ is a zero of $p(x)$

$\Rightarrow p(-3) = 0$

الشرح (بالعربي):

- إذا كان $(x + 3)$ عاملاً.
- فإن جذره $x = -3$.
- $\therefore p(-3) = 0$

EXPLANATION (ARABIC)

1- مجموعة الأصفار (Real Zeros): قيم x التي تجعل $f(x) = 0$.

2- حل المعادلة $f(x) = 0$ لإيجاد الأصفار الحقيقية.

3- الأصفار الحقيقية هي نقاط تقاطع المنحنى مع محور x (x-axis).

4- نختبر كل اختيار بالتعويض أو التحليل لإيجاد الأصفار.

5- الدالة الثابتة لا تساوي صفراً إذا كانت $c \neq 0$.

6- إذا كان $(x + 3)$ عاملاً، فـ $x = -3$ يجعلها صفراً.

KEY MATH IDEAS USED

- To find real zeros, set $f(x) = 0$ and solve.
- If $x^2 = a$ and $a < 0$, there is no real solution.
- Real zeros are the x-intercepts of the graph.
- Test each option by substitution or factoring.
- A constant function $f(x) = c$ has no real zeros when $c \neq 0$.
- If $(x + 3)$ is a factor, then $x = -3$ is a zero.

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SET OF REAL ZEROS OF POLYNOMIAL FUNCTIONS

11 If -7 is one of the zeros of the function p, where $p(x) = 2x^2 + 6x + c$ and C is a constant, find C and the set of real zeros of p.

Find C

Since -7 is a zero, then $p(-7) = 0$

$2(-7)^2 + 6(-7) + c = 0$

$98 - 42 + c = 0$

$56 + c = 0$

$c = -56$

Find the set of real zeros

$p(x) = 2x^2 + 6x - 56$

$2x^2 + 6x - 56 = 0$

$x^2 + 3x - 28 = 0$

$(x - 4)(x + 7) = 0$

Set of real zeros = $\{4, -7\}$

Explanation (Arabic):

بما أن -7 صفر للدالة، نعوض $x = -7$ في $p(x)$ ونجعل الناتج يساوي صفرًا لإيجاد c، ثم نحل المعادلة التربيعية الناتجة بتحليلها إلى عوامل للحصول على مجموعة الأصفار الحقيقية.

المفاهيم المستخدمة:

- Zero of a function: $p(x) = 0$
- Factoring quadratic equations
- Set of real zeros

12 If 3 is one of the zeros of the function g, where $g(x) = x^2 - (k - 5)x - 5k$ and k is a constant, find k and the set of real zeros of g.

Find k

Since 3 is a zero, then $g(3) = 0$

$3^2 - (k - 5)(3) - 5k = 0$

$9 - 3k + 15 - 5k = 0$

$24 - 8k = 0$

$-8k = -24$

$k = 3$

Find the set of real zeros

Substitute $k = 3$ in $g(x)$

$g(x) = x^2 - (3 - 5)x - 5(3)$

$g(x) = x^2 + 2x - 15$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

Set of real zeros = $\{-5, 3\}$

Explanation (Arabic):

بما أن 3 صفر للدالة، نعوض $x = 3$ في $g(x)$ ونجعل الناتج يساوي صفرًا لإيجاد k، ثم نعوض قيمة k في الدالة ونحل المعادلة التربيعية الناتجة بتحليلها إلى عوامل للحصول على مجموعة الأصفار الحقيقية.

المفاهيم المستخدمة:

- Zero of a function: $g(x) = 0$
- Substitution
- Factoring quadratic equations

13 **Tourism:** If the number of daily visitors to watch short films for tourism in Egypt is given by the function $p(t) = -3t^2 + 18t + 165$, where t is the number of days since the film was released. On which day will the number of visitors reach zero?

Egypt is one of the most important tourism destinations in the world, thanks to its unique civilization and attractive landmarks.

Solution:

We set $p(t) = 0$ to find the days when the number of visitors is zero.

$p(t) = -3t^2 + 18t + 165$

$-3t^2 + 18t + 165 = 0$

$-3(t^2 - 6t - 55) = 0$

$-3(t - 11)(t + 5) = 0$

$t - 11 = 0$ or $t + 5 = 0$

$t = 11$ or $t = -5$

We reject -5 because t cannot be negative.

Day = (11)

Set of real zeros $\{-5, 11\}$

Explanation (Arabic):

بحيث العدد صفر، نعوض $p(t) = 0$ ثم نحل المعادلة التربيعية بالتحليل إلى عوامل لإيجاد قيم t.

نستبعد القيمة السالبة لأن عدد الأيام لا يمكن أن يكون سالبًا.

المفاهيم المستخدمة:

- Zero of a function: $p(t) = 0$
- Factoring quadratic equations
- Domain of time: $t \geq 0$

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(14) Find the set of real zeros of each of the following:

$$f(x) = x^3 - (-5x^2) - 4x + 20 \quad (1)$$

$\Rightarrow f(x) = 0$

$\Rightarrow x^3 - 5x^2 - 4x + 20 = 0$

$\Rightarrow x(x^2 - 4) - 5(x^2 - 4) = 0$

$\Rightarrow (x^2 - 4)(x - 5) = 0$

$\Rightarrow (x - 2)(x + 2)(x - 5) = 0$

The set of real zeros of f

$\{-2, 2, 5\}$

الشرح (بالعربي):

وضعنا $f(x) = 0$ ثم فصلنا بالتجميع العامل المشترك $(x^2 - 4)$

ثم حللنا الفرق بين مربعين

وأخيرًا حللنا إلى عوامل خطية. $(x^2 - 4) = (x - 2)(x + 2)$

Key Concepts Used:

- Zero of a function: $f(x) = 0$
- Factoring by grouping
- Difference of squares
- Set of real zeros: solutions in $\mathbb{R}$

$$g(x) = x^2 - 2x - 6 \quad (3)$$

$\Rightarrow g(x) = 0$

$\Rightarrow x^2 - 2x - 6 = 0$

The Discriminant (المميز):

$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(-6) = 28 > 0$

The quadratic has two distinct real solutions

$\Rightarrow x = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{2 \pm \sqrt{28}}{2}$

$\Rightarrow x = \dfrac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

The set of real zeros of g

$\{1 - \sqrt{7}, \quad 1 + \sqrt{7}\}$

الشرح (بالعربي):

بما أن $\Delta > 0$ فلعددالة حلول حقيقيان مختلفان.

استخدمنا قانون الجذر التربيعي لإيجاد جذري المعادلة التربيعية.

$$p(x) = 2x^3 - x^2 - 8x + 4 \quad (2)$$

$\Rightarrow p(x) = 0$

$\Rightarrow 2x^3 - x^2 - 8x + 4 = 0$

$\Rightarrow 2x^3 - 8x - x^2 + 4 = 0$

$\Rightarrow 2x(x^2 - 4) - (x^2 - 4) = 0$

$\Rightarrow (x^2 - 4)(2x - 1) = 0$

$\Rightarrow (x - 2)(x + 2)(2x - 1) = 0$

$\Rightarrow x - 2 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad 2x - 1 = 0$

$\Rightarrow x = 2 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = \dfrac{1}{2}$

The set of real zeros of p

$\{-2, \dfrac{1}{2}, 2\}$

الشرح (بالعربي):

قمنا بالتجميع عامل مشترك $(x^2 - 4)$ ، ثم حللنا الفرق بين مربعين

$(x^2 - 4) = (x - 2)(x + 2)$ الحصلة $(x - 2)(x + 2)$

حصلنا على الأصفار الحقيقية.

LESSON EVALUATION

CONCEPT CHECK First

1 Which of the following values of x makes the rational expression $\frac{x-1}{x+2}$ undefined ?

(a) 1 (b) 2 (c) -1 (d) -2 ✓

Solution:
Undefined when the denominator = 0.
$x+2=0 \Rightarrow x=-2$ $x=-2$

الشرح (بالعربي):
تكون الدالة الكسرية غير معرفة عندما يكون المقام مساويًا للصفر.
$x+2=0 \Rightarrow x=2-0 \Rightarrow x=-2$.
إذن القيمة الصحيحة هي $x=-2$.

2 Which of the following values of x makes the function $f(x)=\frac{x+3}{x-2}$ equal to zero ?

(a) -3 ✓ (b) 3 (c) -2 (d) 2

Solution:
Set the numerator equal to zero.
$x+3=0 \Rightarrow x=-3$ $x=-3$

الشرح (بالعربي):
لكي تكون الدالة الكسرية مساوية للصفر، يجب أن يكون
البسط مساويًا للصفر والمقام لا يساوي صفرًا.
$x+3=0 \Rightarrow x=-3$.
وعند $x=-3$ لا لا يساوي المقام صفرًا، إذن القيمة صحيحة.

3 What is the domain of the function g where $g(x)=\frac{x^2-x-2}{x^2+4}$?

(a) $R-\{-2\}$ (b) $R-\{2\}$ (c) $R-\{-4\}$ (d) R ✓

Solution:
$x^2+4>0$ for all real x $(x^2 \geq 0)$
$\Rightarrow$ The domain is R.

الشرح (بالعربي):
كي دائما أكبر من صفر لأي حقيقية لـ x^2+4.
إذن لا توجد قيم تجعل المقام صفرًا، ومجال الدالة هو R. .

4 What is the domain of the function f where $f(x)=\frac{x^2-25}{7}$?

(a) R ✓ (b) $\{7\}$ (c) $R-\{7\}$ (d) $\{-5,5\}$

Solution:
Denominator is the constant $7 \neq 0$ for all x.
$\Rightarrow$ The domain is R.

الشرح (بالعربي):
المقام ثابت ويساوي 7 وهو لا يساوي صفرًا لأي قيمة x.
إذن مجال الدالة هو مجموعة الأعداد الحقيقية R. .

5 What is the set of real zeros of f where $f(x)=\frac{x^2-2x}{x^2+2x}$?

(a) $\{0\}$ (b) $\{2\}$ ✓ (c) $\{-2\}$ (d) $\{2,0\}$

Solution:
$x^2-2x=0 \Rightarrow x(x-2)=0 \Rightarrow x=0$ or $x=2$.
$x^2+2x=x(x+2)=0 \Rightarrow x \neq 0$ and $x \neq -2$.
Thus, the only real zero is $x=2$. $\{2\}$

الشرح (بالعربي):
أصفار الكون عندما يكون البسط صفرًا والمقام لا يساوي صفرًا.
المقرًا: $x=0$ لكن $\Rightarrow x=0$ $x^2-2x=x(x+2)=0 \Rightarrow$
أصفار المقام: $x \neq 0, -2$ $\Rightarrow x^2+2x=x(x+2)=0$.
إذن الجذر الحقيقي الوجيد هو $x=2$.

6 If the domain of f where $f(x)=\frac{2x}{x-a}$ is equal to $R-\{-2\}$, what is the value of a ?

(a) 1 (b) -1 (c) 2 (d) -2 ✓

Solution:
$x-a=0$ is excluded from the domain $\Rightarrow x=a$.
Given $R-\{-2\}$, the excluded value is $x=-2$.
Hence, $a=-2$. $a=-2$

الشرح (بالعربي):
القيمة التي تجعل المقام صفرًا هي $x=a$.
وبما أن المجال بستثني -2 إذن $a=-2$.

7 If the set of real zeros of f where $f(x)=\frac{x^2-x+a}{x+2}$ is $\{3\}$, what is the value of a ?

(a) -6 ✓ (b) -2 (c) 3 (d) 6

Solution:
Zeros of the rational function = zeros of numerator − zeros of denominator.
Denominator zero: $x+2=0 \Rightarrow x=-2$.
Numerator: $x^2-x+a=0$ must have only one real zero: $x=3$ (and $x \neq -2$).
Substitute $x=3$ into the numerator:
$3^2-3+a=0 \Rightarrow 9-3+a=0 \Rightarrow a=-6$. $a=-6$

Check:
$\frac{x^2-x-6}{x+2}=\frac{(x-3)(x+2)}{x+2}=x-3 \quad (x \neq -2)$
$x-3=0 \Rightarrow x=3$
At $x=-2$ the expression is undefined.
So, the only real zero is $\{3\}$ ✓

الشرح (بالعربي):
أصفار الدالة الكسرية = أصفار البسط − أصفار المقام.
أصفار المقام: $x=-2$.
لكي يكون الجذر الحقيقي الوجيد 3، نعوض $x=3$ في البسط:
$3^2-3+a=0 \Rightarrow 9-3+a=0 \Rightarrow a=-6$. كيبد ر التحقق إن الجذر الوحيد هو 3

Common Mistake
Some students write the domain without finding the values that make the denominator zero.
Wrong: $x=-1$
Correct: $\{-1\}$

Practice for You (from self-assessment)

① Find the values that make the denominator zero:
$\frac{3x-6}{x^2-x-2}$
Answer: $\{-1, 2\}$

② Find the values that make the denominator zero:
$\frac{x^2+4x+3}{x-1}$
Answer: $\{1\}$

Important Note
Do not simplify by canceling factors before finding the values that make the denominator zero.
Find the values first, then simplify if needed.

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LESSON EVALUATION

Detect the Error

8 Each of Aml and Majdi simplified the function f in simplest form as shown.

$$f(x) = \frac{4x + 12}{x^2 - x - 12}$$

Find the domain of f.
Who is correct? And why?

Wrong Solution

$$\therefore f(x) = \frac{4x + 12}{x^2 - x - 12}$$

$$\therefore f(x) = \frac{4(x + 3)}{(x + 3)(x - 4)}$$

$$\therefore f(x) = \frac{4}{x - 4}$$

$$\therefore D_f = \mathrm{R} - \{4\}$$

Correct Solution

$$\therefore f(x) = \frac{4x + 12}{x^2 - x - 12}$$

$$\therefore f(x) = \frac{4(x + 3)}{(x + 3)(x - 4)}$$

$$\therefore D_f = \mathrm{R} - \{-3, 4\}$$

$$\therefore f(x) = \frac{4}{x - 4}$$

Aml is correct

Majdi is wrong

The domain before simplification is not the same as after simplification.

Apply the Concepts

9 Find the simplest form of the function f and state its domain in each of the following:

1

$$f(x) = \frac{2x - 5}{5 - 2x}$$

$$f(x) = \frac{2x - 5}{-(2x - 5)}$$

$$D_f = \mathrm{R} - \left\{\frac{5}{2}\right\}$$

$$f(x) = -1$$

Domain: $\mathrm{R} - \left\{\frac{5}{2}\right\}$

الشرح (بالعربي):
$5 - 2x = -(2x - 5)$
ثم اختزل العامل المشترك $(2x - 5)$
مع تذكر أن $x \neq \frac{5}{2}$.

2

$$f(x) = \frac{x - 4}{x^2 - 16}$$

$$f(x) = \frac{x - 4}{(x - 4)(x + 4)}$$

$$D_f = \mathrm{R} - \{4, -4\}$$

$$f(x) = \frac{1}{x + 4}$$

Domain: $\mathrm{R} - \{4, -4\}$

الشرح (بالعربي):
حلل المقام إلى عوامل مشتركة
ثم اختزل العامل المشترك $(x - 4)$
مع تذكر القيم غير المسموح بها.

3

$$f(x) = \frac{x^2 - 3x + 2}{3x - 3}$$

$$f(x) = \frac{(x - 1)(x - 2)}{3(x - 1)}$$

$$D_f = \mathrm{R} - \{1\}$$

$$f(x) = \frac{x - 2}{3}$$

Domain: $\mathrm{R} - \{1\}$

الشرح (بالعربي):
حلل البسط إلى $(x - 1)(x - 2)$
واختزل العامل المشترك $(x - 1)$
مع تذكر أن $x \neq 1$.

4

$$f(x) = \frac{6x + 12}{2x^2 + 7x + 6}$$

$$f(x) = \frac{6(x + 2)}{(2x + 3)(x + 2)}$$

$$D_f = \mathrm{R} - \left\{-2, -\frac{3}{2}\right\}$$

$$f(x) = \frac{6}{2x + 3}$$

Domain: $\mathrm{R} - \left\{-2, -\frac{3}{2}\right\}$

الشرح (بالعربي):
حلل المقام إلى $(2x + 3)(x + 2)$
واختزل العامل المشترك $(x + 2)$
مع تذكر أن $x \neq -2$ و $x \neq -\frac{3}{2}$.

EXPLANATION (ARABIC)

- نحلل البسط والمقام إلى عوامل.
- نختزل العوامل المشتركة للحصول على أبسط صورة.
- قبل الاختزال، قيم المقام التي تجعله = 0 نستبعد من المجال.
- المجال هو جميع الأعداد الحقيقية ما عدا القيم المستبعدة.
- أبسط صورة لا تغير من قيمة الدالة فقط تجعلها أبسط.

KEY MATH IDEAS USED

- Factoring polynomials
- Cancelling common factors
- Domain of a rational function: denominator $\neq 0$
- Simplest form of a rational expression
- Domain is the set of all real numbers except excluded values

LESSON EVALUATION

5 $f(x) = \dfrac{16 + 16x + 3x^2}{x^2 - 3x - 28}$

$$f(x) = \frac{(x+4)(3x+4)}{(x+4)(x-7)}$$

$$D_f = \mathbb{R} - \{-4 \ , \ 7\}$$

$$f(x) = \frac{3x+4}{x-7}$$

الشرح (بالعربي):
حللنا البسط والمقام، ثم اختزلنا العامل المشترك $(x+4)$.
مجال الدالة هو جميع الأعداد الحقيقية ما عدا -4 , 7
لأنهما يجعلان المقام الأصلي = صفر.

6 $f(x) = \dfrac{4x^2 - 8x - 5}{15 - x - 2x^2}$

$$f(x) = \frac{(2x+1)(2x-5)}{-(2x^2+x-15)}$$

$$= \frac{(2x+1)(2x-5)}{-(2x-5)(x+3)}$$

$$D_f = \mathbb{R} - \left\{-3 \ , \ \frac{5}{2}\right\}$$

$$f(x) = \frac{2x+1}{-(x+3)}$$

الشرح (بالعربي):
حللنا البسط والمقام إلى عوامل مشتركة،
ثم اختزلنا العامل المشترك $(2x-5)$.
مجال الدالة هو جميع الأعداد الحقيقية ما عدا
القيم التي تجعل المقام الأصلي = صفر.

10 Find the set of zeros of each function f in the simplest form as follows:

1 $f(x) = \dfrac{x+2}{x-2}$

أصفار البسط : $\{-2\}$
أصفار المقام : $\{2\}$

أصفار الدالة
$\Rightarrow \{-2\} - \{2\}$
$\Rightarrow \{-2\}$

الشرح (بالعربي):
أصفار الدالة هي أصفار البسط فقط مع استبعاد أصفار المقام لأنه لا يمكن أن تكون قيمة تجعل المقام = صفر.

2 $f(x) = \dfrac{x^2 - 9}{x+3}$

أصفار البسط : $\{3 \ , -3\}$
أصفار المقام : $\{-3\}$

أصفار الدالة
$\Rightarrow \{3 \ , -3\} - \{-3\}$
$\Rightarrow \{3\}$

الشرح (بالعربي):
أصفار الدالة هي أصفار البسط فقط مع استبعاد أصفار المقام لأنه لا يمكن أن تكون قيمة تجعل المقام = صفر.

3 $f(x) = \dfrac{2x-6}{x^2 - x - 6}$

$$f(x) = \frac{2(x-3)}{(x+2)(x-3)}$$

أصفار البسط : $\{3\}$
أصفار المقام : $\{-2 \ , \ 3\}$ أو $\{-3\}$

أصفار الدالة
$\Rightarrow \{3\} - \{-2 \ , \ 3\}$
$= \varnothing$

الشرح (بالعربي):
حللنا البسط والمقام، ثم اختزلنا العامل المشترك $(x-3)$.
القيمة 3 تصبح غير مسموح بها (لأنها تجعل المقام الأصلي = صفر)، فتكون مجموعة الأصفار فارغة.

4 $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 + x - 6}$

$$f(x) = \frac{(2x-1)(x+3)}{(x-2)(x+3)}$$

أصفار البسط : $\{\frac{1}{2} \ , -3\}$
أصفار المقام : $\{2 \ , -3\}$

أصفار الدالة
$\Rightarrow \{\frac{1}{2} \ , -3\} - \{2 \ , -3\}$
$= \{\frac{1}{2}\}$

الشرح (بالعربي):
حللنا البسط والمقام، ثم اختزلنا العامل المشترك $(x+3)$.
أصفار الدالة هي أصفار البسط فقط مع استبعاد أصفار المقام.

ملخص الشرح (بالعربي):

- نحلل البسط والمقام إلى عوامل.
- نختزل العوامل المشتركة للحصول على أبسط صورة.
- قبل الاختزال، نحدد القيم التي تجعل المقام الأصلي = صفر ونستبعدها من المجال.
- أصفار الدالة هي أصفار البسط فقط (بعد الاختزال) مع استبعاد أصفار المقام.

Key Math Ideas Used:

- Factoring polynomials
- Cancelling common factors
- Domain of a rational function: denominator ≠ 0
- Zeros of a function: numerator = 0 (and denominator ≠ 0)
- Set notation and set difference

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LESSON EVALUATION

Review all solutions and convert terms to English with a simple explanation in Arabic

5 $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 7x + 12}$

(a) Factor the numerator

$x^2 - 3x + 2$
$= (x - 1)(x - 2)$ ← $\{1 , 2\}$

(b) Factor the denominator

$x^2 - 7x + 12$
$= (x - 4)(x - 3)$ ← $\{3 , 4\}$

(c) The zeros of the function

$\Rightarrow \{1 , 2\} - \{3 , 4\} = \{1 , 2\}$

الشرح (بالعربي):
خللنا البسط والمقام، ثم اختزلنا العامل المشترك.
مجال الدالة هو جميع الأعداد الحقيقية ما عدا 3 , 4
لأنهما يجعلان المقام الأصلي = صفر.

6 $f(x) = \dfrac{4x^2 - 4x - 3}{9 - 3x - 2x^2}$

(a) Factor the numerator

$4x^2 - 4x - 3$
$= (2x + 1)(2x - 3)$ $\left\{-\frac{1}{2} , \frac{3}{2}\right\}$

(b) Factor the denominator

$9 - 3x - 2x^2$
$= -(2x^2 + 3x - 9)$
$= -(2x - 3)(x + 3)$ $\left\{\frac{3}{2} , -3\right\}$

(c) The zeros of the function

$\Rightarrow \left\{-\frac{1}{2} , \frac{3}{2}\right\} - \left\{\frac{3}{2} , -3\right\}$
$= \left\{-\frac{1}{2}\right\}$

الشرح (بالعربي):
خللنا البسط والمقام إلى عوامل مشتركة،
ثم اختزلنا العامل المشترك $(2x - 3)$.
أصفار الدالة هي أصفار البسط بعد الاختزال
مع استبعاد أصفار المقام (لأنها تجعل المقام = صفر).

11 If the domain of the function $f(x) = \dfrac{x + 6}{x^2 - ax + 36}$ is $R - \{6\}$, find the value of a

$D_f = R - \{6\}$

$x^2 - ax + 36 = 0$ ← when $x = 6$
$36 - 6a + 36 = 0$
$-6a = -72$
$a = 12$

الشرح (بالعربي):
حتى يكون 6 خارج المجال،
يجب أن يجعل المقام = صفر.
نعوض $x = 6$ في المقام
ونحل لإيجاد قيمة a.

12 If $\dfrac{x + a}{x - b}$, $f(2)$ is undefined, and $f(5) = 2$, find the values of a and b

(a) Find b

$f(2)$ غير معرفة

$D_f = R - \{2\}$

$x - b = 0$
$2 - b = 0$
$-b = -2$
$b = 2$

الشرح (بالعربي):
$f(2)$ غير معرفة يعني
المقام = صفر عند $x = 2$
فنحل $x - b = 0$
لإيجاد قيمة b.

(b) Find a

$f(x) = \dfrac{x + a}{x - 2}$

- $f(5) = 2$
- $\dfrac{5 + a}{5 - 2} = 2$
- $5 + a = 6$

$a = 1$

الشرح (بالعربي):
نعوض $x = 5$ في الدالة ونحل المعادلة لإيجاد a.

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LESSON EVALUATION

Review all solutions and convert terms to English with a simple explanation in Arabic

13 If $f(x)=\frac{2x^2+7x-15}{2x-3}$, find the set of solutions to the equation $f(x)=10$ in $\mathbb{R}$.

Solution Steps:

- $f(x)=10$
- $\frac{2x^2+7x-15}{2x-3}=10$
- $2x^2+7x-15=10(2x-3)$
- $2x^2+7x-20x-15+30=0$
- $2x^2-13x+15=0$
- $(2x-3)(x-5)=0$

Check the denominator:

$2x-3\neq 0 \Rightarrow x\neq 3/2$

$x=5$

Solution set = { 5 }

(الشرح بالعربي):

- حولنا البسط والمقام إلى عوامل، ثم حولنا المعادلة إلى معادلة.
- حللنا المعادلة التربيعية إلى عوامل.
- في النهاية استبعدنا القيمة التي تجعل المقام = صفر.

14 If $g(x)=\frac{x^2-5x+6}{x-2}$, where $x\in\mathbb{R}$, find the set of solutions to the equation $g(x)=-1$.

Solution Steps:

- $g(x)=-1$
- $\frac{x^2-5x+6}{x-2}=-1$
- $x^2-5x+6=-1(x-2)$
- $x^2-5x+x+6-2=0$
- $x^2-4x+4=0$

Important check

$(x-2)(x-2)=0$

$x=2$

x = 2 is NOT in the domain.

Solution set = ∅

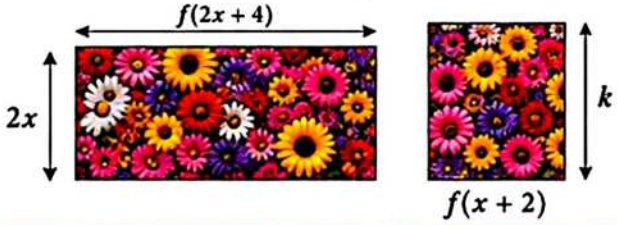
(الشرح بالعربي):

- حولنا المعادلة إلى معادلة تربيعية بعد حذف المقام بضرب طرفي المعادلة في $(x-2)$.
- لاحظ أن $x=2$ يجعل المقام = صفر، لكن حل المعادلة يساوي $x=2$ أيضاً، فهو حل مقبول.

15 Rectangle:

In the figure, the rectangles are similar, and k is a constant such that the function f in x has the same area. In the figure, find $f(x)$ in simplest form, then find the value of k in each of the following cases:

1. The value of k when $x=2$
2. The value of k when $x=4$



Area of the first rectangle = Area of the second rectangle

- $2x(2x+4)=k(x+2)$
- $4x^2+8x=kx+2k$
- $f(x)=4x^2+x(8-k)-2k$

$x=2$

$4(2)^2+8(2)-2k-2k=0$

$-4k=-32$

$k=8$

x = 4

$64+32-4k-2k=0$

$96-6k=0$

$k=16$

k = 16

(الشرح بالعربي):

- لإيجاد قيمة k نعوض عن قيم x المطلوبة في معادلة المساحات ثم نحل المعادلة الناتجة.

Verified: x=2 ⇒ k=8 | x=4 ⇒ k=16

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16 Open Question: Write a rational function whose domain is $R - \{4, -2\}$.

Solution Steps:

$$f(x) = \frac{2x+4}{x^2-2x-8}$$

$$f(x) = \frac{2(x+2)}{(x+2)(x-4)}$$

Simplify:

$$f(x) = \frac{2}{x-4}$$

Math Terminology Used:

- Domain: set of all real numbers that make the denominator $\neq 0$.
- Factorization: writing an expression as a product of linear factors.
- Canceling common factors.
- Simplest form of a rational function.

Conclusion:

$D_f = R - \{4, -2\}$

(الشرح بالعربي:)

حللنا المقام إلى عوامل فحصلنا على $(x+2)(x-4)$ ، إذن القيم التي تجعل المقام صفر هي $x = -2$ أو $x = 4$ ، لذلك المجال هو $R - \{4, -2\}$.
اختزلنا العامل المشترك $(x+2)$ في البسط والمقام لنحصل على أبسط صورة.
إذن الدالة المطلوبة هي $f(x) = \frac{2}{x-4}$.

---

17 If the volume of box A is $(x^3 - 2x^2 - 3x)$ cubic units, and the volume of box B is $(x^3 + 2x^2 - 15x)$ cubic units, what is the ratio of the volume of A to the volume of B in simplest form? Can this ratio be $\frac{1}{2}$? Why?

Solution Steps:

The ratio:

$$\frac{\text{Volume of } A}{\text{Volume of } B} = \frac{x^3 - 2x^2 - 3x}{x^3 + 2x^2 - 15x}$$

Factor:

$$= \frac{x(x^2-2x-3)}{x(x^2+2x-15)} = \frac{x(x+1)(x-3)}{x(x-3)(x+5)}$$

Simplify:

$$= \frac{x+1}{x+5}$$

Domain:

$R - \{0, 3, -5\}$

For the ratio to be $\frac{1}{2}$:

$$\frac{x+1}{x+5} = \frac{1}{2}$$

$$2(x+1) = x+5 \Rightarrow x = 3$$

Math Terminology Used:

- Ratio of two expressions.
- Factorization using special products.
- Canceling common factors.
- Domain: denominator $\neq 0$.
- Checking if a value belongs to the domain.

Can the ratio be $\frac{1}{2}$?

No, because $x = 3 \notin R - \{0, 3, -5\}$
(3 is not allowed).
So, $\frac{1}{2}$ is not possible.

(الشرح بالعربي:)

فككنا كل من البسط والمقام:

$x^3 - 2x^2 - 3x = x(x+1)(x-3)$

$x^3 + 2x^2 - 15x = x(x-3)(x+5)$

ثم اختزلنا العاملين المشتركين x و $(x-3)$ لنحصل على $\frac{x+1}{x+5}$.
المجال هو جميع الأعداد الحقيقية ما عدا القيم التي تجعل المقام صفرًا:
$x \neq 0, 3, -5$.
عند مساواة النسبة بـ $\frac{1}{2}$ وجدنا $x = 3$ لكنها غير مسموح بها في المجال،
إذا النسبة $\frac{1}{2}$ غير ممكنة .

LESSON EVALUATION

LESSON 2 - 4

المجال المشترك وتساوي دوال الكسور الجبرية النسبية
(Common Domain and Equivalent Rational Algebraic Functions)

First: Concepts Measured

▶ **Choose the correct answer from the given choices:**

1 If $g(x) = \dfrac{-1}{x-2}$, what is the common domain of the two functions g and f ?

(a) $R - \{2\}$ (b) $R - \{-2\}$ (c) $R - \{-2, 2\}$ ✔ (d) R

Explanation (Arabic):
مجال $g(x)$ هو $R - \{2\}$ لأن المقام لا يساوي صفرًا عند $x = 2$.
ومجال $f(x)$ هو $R - \{-2\}$ لأن المقام لا يساوي صفرًا عند $x = -2$.
إذن المجال المشترك هو استيعاد النقطتين 2 و 2 − معًا.

$D_g = R - \{2\}$
$D_f = R - \{-2\}$

Common Domain
$R - \{-2, 2\}$

2 If $f(x) = \dfrac{x}{x(x-3)}$, $g(x) = \dfrac{1}{x-3}$, which of the following sets represents the values of x for which $f(x) = g(x)$?

(a) $R - \{3\}$ (b) $R - \{0\}$ (c) $R - \{0, 3\}$ ✔ (d) R

$D_g = R - \{3\}$
$D_f = R - \{0, 3\}$

Common Domain
$R - \{0, 3\}$

=

Explanation (Arabic):
مجال f يستبعد $x = 0$ و $x = 3$ لأن المقام $x(x-3)$ يساوي صفرًا عندهما.
ومجال g يستبعد $x = 3$ فقط.
إذن القيم المشتركة هي استبعاد 0 و 3 معا.

3 If $f(x) = \dfrac{x}{x-k}$, $g(x) = \dfrac{-2}{x+3}$, and the common domain of the two functions is $R - \{-3\}$, what is the value of k ?

(a) 3 (b) -3 ✔ (c) 0 (d) -2

$D_f = R - \{k\}$

Common Domain
$R - \{-3\}$

$x - k = 0 \Rightarrow x = k$

$k = -3$

Key Math Terminology Used:

- **Domain:** set of all real numbers that make the denominator $\neq 0$.
- **Common Domain:** intersection of domains of the functions.
- **Excluded value:** value that makes the denominator zero.
- **Equivalent rational functions:** functions equal on their common domain.

ملخص (بالعربي):

- نحدد قيم x التي تجعل المقام صفرًا ونستبعدها من المجال.
- المجال المشترك هو تقاطع مجالي الدالتين.
- في (2) نساوي الدالتين ونعمل في المجال المشترك فقط.
- في (3) نساوي مجالي الدالتين ونستنتج قيمة k .

المجال المشترك وتساوي دوال الكسور الجبرية النسبية
(Common Domain and Equivalent Rational Algebraic Functions)

4 Spot the Mistake:

Each of the arguments below made a claim and justification in response to the following question:

If $f(X) = \frac{x^2-1}{x^2-4x-5}$, $g(X) = \frac{x^2-x}{x^2-5x}$, is $f = g$?

Iyman's Answer (Incorrect):

- $f(x) = \frac{(x-1)(x+1)}{(x+1)(x-5)}$ (cancel)
- $\therefore f(x) = \frac{x-1}{x-5}$
- $g(x) = \frac{x(x-1)}{x(x-5)}$ (cancel)
- $g(x) = \frac{x-1}{x-5}$
- $\therefore f(x) = g(x)$
- $\therefore f = g$

Doaa's Answer (Correct):

- $f(x) = \frac{(x-1)(x+1)}{(x+1)(x-5)}$ (factor)
- $D_f = R - \{-1, 5\}$, $f(x) = \frac{x-1}{x-5}$
- $g(x) = \frac{x(x-1)}{x(x-5)}$ (factor)
- $D_g = R - \{0, 5\}$, $g(x) = \frac{x-1}{x-5}$
- $D_f \neq D_g$, $f(x) = g(x)$
- $\therefore f \neq g$

Which of their answers is correct? Why?

Iyman's Answer (Incorrect)

Because he must find the domain of both functions before canceling common factors.

Doaa's Answer (Correct)

5 Apply Concepts:

Find the common domain of the two functions f and g in each of the following:

$$f(x) = \frac{x+1}{x-3}$$

Set the denominator = 0

$x - 3 = 0 \Rightarrow x = 3$

$D_f = R - \{3\}$

$$g(x) = \frac{x-1}{x-2}$$

Set the denominator = 0

$x - 2 = 0 \Rightarrow x = 2$

$D_g = R - \{2\}$

Math Terminology Used:

- Domain: set of all real numbers that make the denominator $\neq 0$.
- Common domain: intersection of the domains of the functions.
- Excluded values: values that make the denominator zero.

Common domain of the two functions f and g

$$R - \{2, 3\}$$

الشرح (بالعربي)

- في (4) المفترض بحدد مجال كل دالة قبل الاختزال.
- نحلل المقام إلي عوامل (Factorization).
- نستبعد الأعداد التي تجعل المقام صفرًا (Excluded values).
- نقارن المجالين ؛ إذا اختلفا فالدالتان غير متكافئتين رغم تساوي صيغتهما المبسطة.
- في (5) نجد مجال كل دالة بوضع المقام = 0 ثم نستبعد هذه القيم.
- المجال المشترك هو تقاطع المجالين.

Key Math Terminology Used:

- Factorization: rewriting an expression as a product of factors.
- Domain: all real numbers that make the denominator nonzero.
- Common domain: intersection of the domains of the functions.
- Excluded values: values that make the denominator zero.
- Equivalent rational functions: functions that are equal on their common domain.

المجال المشترك للدوال الجبرية النسبية
(Common Domain and Equivalent Rational Algebraic Functions)

2 Find the common domain of the two functions f and g.

$$f(x) = \frac{x^2 - x - 20}{x^2 - 3x - 10}$$

Set the denominator = 0 :

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$$\Rightarrow \quad x = -2 \quad \text{or} \quad x = 5$$

$$D_f = R - \{-2, 5\}$$

$$g(x) = \frac{x^2 - 3x - 10}{x^2 - x - 6}$$

Set the denominator = 0 :

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$\Rightarrow \quad x = -2 \quad \text{or} \quad x = 3$$

$$D_g = R - \{-2, 3\}$$

Common domain $R - \{-2, 3, 5\}$

Explanation (Arabic):

- حددنا قيم x التي تجعل المقام = 0 في كل دالة.
- هذه القيم تُستبعد من المجال.
- المجال المشترك هو تقاطع مجالي الدالتين.

Math Terminology Used:

- Denominator: the expression below the fraction bar.
- Domain: all real numbers except those making the denominator = 0.
- Common domain: intersection of the domains of the functions.

Why Doaa's answer is correct?

- لم تلغ العوامل المشتركة قبل تحديد المجال.
- استبعدت جميع القيم التي تجعل المقام = 0 في كل دالة.

6 Find the common domain of the functions f, g, and k.

$$f(x) = \frac{x+2}{3}$$

$$g(x) = \frac{3}{x^2 - 9}$$

$$k(x) = \frac{5x}{x^2 - 3x}$$

$$k(x) = \frac{5x}{x(x-3)} = \frac{5}{x-3} \rightarrow D_k = R - \{0, 3\}$$

المقام = 0 عندما $x = 3$ أو $x = 0$

$$g(x) = \frac{3}{x^2 - 9} = \frac{3}{(x-3)(x+3)} \rightarrow D_g = R - \{-3, 3\}$$

المقام = 0 عندما $x = 3$ أو $x = -3$

$$f(x) = \frac{x+2}{3} = \frac{1}{3}(x+2) \rightarrow D_f = R$$

المقام ثابت (3) ≠ 0 دائما

Common domain of f, g, k $R - \{-3, 0, 3\}$

Explanation (Arabic):

- حددنا قيم x التي تجعل المقام = 0 في كل دالة.
- استبعدنا هذه القيم من كل مجال.
- المجال المشترك هو تقاطع المجالات الثلاثة.

Key Math Terminology Used:

- Factorization: writing an expression as a product.
- Canceling common factors.
- Excluded values: values that make the denominator zero.
- Common domain: intersection of three domains.

Summary in Arabic:

- في (2) استبعدنا: $-2, 3, 5$
- في (6) استبعدنا: $-3, 0, 3$
- المجال المشترك هو مجموعة الأعداد الحقيقية بعد استبعاد القيم المستبعدة.

LESSON REVIEW

LESSON 2 - 4

أثبت أن $f = g$: في كل مما ياتي :

Prove that $f = g$ in each of the following:

1

$$g(x) = \frac{3x}{3x^2 - 12}$$

$$f(x) = \frac{x}{x^2 - 4}$$

Solution Steps:

- $g(x) = \frac{3x}{3(x^2 - 4)}$

$$= \frac{3x}{3(x-2)(x+2)}$$

$$= \frac{x}{(x-2)(x+2)}$$

Domain of g : $D_g = R - \{-2, 2\}$

(Denominator $\neq 0 \Rightarrow x \neq -2, 2$)

Solution Steps:

$$f(x) = \frac{x}{(x-2)(x+2)}$$

Domain of f : $D_f = R - \{-2, 2\}$

(Denominator $\neq 0 \Rightarrow x \neq -2, 2$)

Compare Domains: $D_f = D_g$

Compare Functions: $f(x) = g(x)$

$\therefore f = g$

Math Terminology

- Factorization
- Canceling common factors
- Domain (Excluded values)
- Equivalent rational functions

Brief Explanation (Arabic)

حللنا المقامات إلى عوامل خطية، وألغينا العامل المشترك 3 في الدالة g. وجدنا أن مجالي الدالتين متساويان، والدالتان متساويتان في تبسيطهما.

2

$$g(x) = \frac{x}{x-2}$$

$$f(x) = \frac{x^2 - 2x}{x^2 - 4x + 4}$$

Solution Steps:

- $g(x) = \frac{x}{x-2}$

Domain of g : $D_g = R - \{2\}$

(Denominator $\neq 0 \Rightarrow x \neq 2$)

Compare Domains: $D_f = D_g$

$\therefore f = g$

Solution Steps:

- $f(x) = \frac{x(x-2)}{(x-2)(x-2)}$

$$= \frac{x}{x-2}$$

Domain of f : $D_f = R - \{2\}$

(Denominator $\neq 0 \Rightarrow x \neq 2$)

Compare Functions: $f(x) = g(x)$

$\therefore f = g$

Math Terminology

- Factorization
- Canceling common factors
- Domain (Excluded values)
- Equivalent rational functions

Brief Explanation (Arabic)

حللنا البسط والمقام في الدالة f ، ثم ألغينا العامل المشترك $(x - 2)$. مجالي الدالتين متساويان . والتبسيطان متساويان

3

$$g(x) = \frac{6x}{6x + 12}$$

$$f(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$$

Solution Steps:

- $g(x) = \frac{6x}{6(x+2)}$

$$= \frac{x}{x+2}$$

Domain of g : $D_g = R - \{-2\}$

(Denominator $\neq 0 \Rightarrow x \neq -2$)

$g(x) = \frac{x}{x+2}$

$\therefore f = g$

Solution Steps:

- $f(x) = \frac{x(x+2)}{(x+2)(x+2)}$

$$= \frac{x}{x+2}$$

Domain of f : $D_f = R - \{-2\}$

(Denominator $\neq 0 \Rightarrow x \neq -2$)

$f(x) = \frac{x}{x+2}$

$\therefore f = g$

Math Terminology

- Factorization
- Canceling common factors
- Domain (Excluded values)
- Equivalent rational functions

Brief Explanation (Arabic)

حللنا الدالة f إلى عوامل خطية، وألغينا العامل المشترك $(x + 2)$. مجالي الدالتين متساويان والتبسيطان متساويان.

Key Math Terminology Used

- Factorization: rewriting an expression as a product of factors.
- Canceling common factors: dividing numerator and denominator by their common factor.
- Domain: set of all real numbers that make the denominator nonzero.
- Excluded values: values that make the denominator zero.
- Equivalent rational functions: functions that are equal on their common domain.

بالعربي (Summary in Arabic)

- نحلل البسط والمقام إلى عوامل خطية (Factorization).
- نلغي العامل المشترك بين البسط والمقام (Canceling common factors).
- نحدد قيم x التي تجعل المقام 0 = ونستبعدها من المجال (Domain).
- نقارن مجالي الدالتين، فإذا كانا متساويين والتبسيطان متساويين، فإن $f = g$ (Equivalent rational functions).

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LESSON REVIEW

LESSON 2 - 4

المجال المشترك للدوال النسبية
Common Domain and Equivalent Rational Functions

8 For each pair, is $f = g$ or not? And why?

1

$$g(x) = \frac{x^2+2x-3}{x^2+7x+12} \qquad f(x) = \frac{x^2-5x+4}{x^2-16}$$

$$g(x) = \frac{(x-1)(x+3)}{(x+4)(x+3)} \qquad f(x) = \frac{(x-1)(x-4)}{(x-4)(x+4)}$$

$D_g = R - \{-4, -3\}$ $\qquad$ $D_f = R - \{-4, 4\}$

$$g(x) = \frac{x-1}{x+4} \qquad f(x) = \frac{x-1}{x+4}$$

$D_g \neq D_f$ (Domains are different)

$\therefore f \neq g$ (Not equal)

2

$$g(x) = \frac{2x-6}{2x^2-10x+12} \qquad f(x) = \frac{x^2-3x}{x^2-5x+6}$$

$$g(x) = \frac{2(x-3)}{2(x-2)(x-3)} \qquad f(x) = \frac{x(x-3)}{(x-2)(x-3)}$$

$D_g = R - \{2, 3\}$ $\qquad$ $D_f = R - \{2, 3\}$

$D_f = D_g$ (Domains are equal)

$$g(x) = \frac{1}{x-2} \qquad f(x) = \frac{x}{x-2}$$

$f(x) \neq g(x)$ (Functions are not equal)

Math Terminology

- Factorization: writing an expression as a product of factors.
- Canceling common factors: dividing numerator and denominator by their common factor.
- Domain: all real numbers that make the denominator ≠ 0.
- Common domain: the intersection of the domains of the functions.
- Equivalent rational functions: functions that are equal on their common domain.

Brief Explanation (Arabic)

حللنا البسط والمقام إلى عوامل خطية، وألغينا العامل المشترك ثم حددنا قيم x التي تجعل المقام = 0 لاستخراج المجال، ومن ثم قارنا المجالات والدالتين بعد الاختزال.

9 Find the values of x that make $g(x) = f(x)$ for the two functions f and g below:

1

$$g(x) = \frac{1}{x+1} \qquad f(x) = \frac{x-1}{x^2-1}$$

$$g(x) = \frac{1}{x+1} \qquad f(x) = \frac{x-1}{(x-1)(x+1)}$$

$D_g = R - \{-1\}$ $\qquad$ $f(x) = \frac{1}{x+1}$

$D_f = R - \{-1, 1\}$

Values of x, and the common domain of the two functions f and g

$$R - \{-1, 1\}$$

Math Terminology

- Factorization
- Canceling common factors
- Domain (Excluded values)
- Common domain
- Equivalent rational functions

Brief Explanation (Arabic)

حللنا المقام إلى عوامل خطية، وألغينا العامل المشترك، وحددنا القيم التي تجعل المقام = 0، ثم أخذنا تقاطع المجالين لايجاد المجال المشترك حيث الدالتان متساويتان.

Explanation (Arabic)

- في (1–8): بعد الاختزال المجالان مختلفان (3− غير مستبعدة من g ومستبعدة من f و 4 مستبعدة من f فقط)، لذلك $f \neq g$.
- في (2–2): المجالان متساويان $R - \{2, 3\}$ ، ولكن الدالتان بعد الاختزال مختلفتان ($\frac{1}{x-2} \neq \frac{x}{x-2}$) إذا $f \neq g$.
- في (1–9): الدالتان بعد الاختزال متساويتان $\frac{1}{x+1}$ ، والمجال المشترك بين $R - \{-1, 1\}$ و $R - \{-1, 1\}$ هو $R - \{-1, 1\}$

Key Math Terminology Used

- Factorization: rewriting an expression as a product of factors.
- Canceling common factors: dividing numerator and denominator by their common factor.
- Domain: set of all real numbers that make the denominator ≠ 0.
- Common domain: intersection of the domains of the functions.
- Equivalent rational functions: functions that are equal on their common domain.

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LESSON REVIEW

LESSON 2 - 4

المجال المشترك للدوال النسبية
Common Domain of Rational Functions

2

$g(x) = \dfrac{x^2 + x}{x^2 - 3x}$

$g(x) = \dfrac{x(x+1)}{x(x-3)}$

$D_g = R - \{0, 3\}$

x values in the common domain $= R - \{0, 3\}$

$f(x) = \dfrac{x+1}{x-3}$

$f(x) = \dfrac{x+1}{x-3}$

$D_f = R - \{3\}$

x values in the common domain $= R - \{0, 3\}$

Common domain $= R - \{0, 3\} = x$ values in the common domain

3

$g(x) = \dfrac{x+1}{x^2 - 3x - 4}$

$g(x) = \dfrac{x+1}{(x+1)(x-4)}$

$D_g = R - \{-1, 4\}$

$g(x) = \dfrac{1}{x-4}$

$f(x) = \dfrac{x+4}{x^2 - 16}$

$f(x) = \dfrac{x+4}{(x+4)(x-4)}$

$D_f = R - \{-4, 4\}$

$f(x) = \dfrac{1}{x-4}$

Common domain $= R - \{-4, -1, 4\} = x$ values in the common domain

Math Terminology

- Factorization: writing an expression as a product of factors.
- Cancelling common factors: dividing numerator and denominator by their common factor.
- Domain (Excluded values): all real numbers that make the denominator $\neq 0$.
- Common domain: intersection of the domains of the functions.
- Equivalent rational functions: functions that are equal on their common domain.

شرح مبسط (بالمصطلحات الرياضية)

(2) حللنا المقامات إلى عوامل خطية (Factorization)، ثم اخترلنا العوامل المشتركة بين البسط والمقام (Cancelled common factors). حذدنا قيم x التي تجعل المقام = 0, (Excluded values) فوجدنا مجال كل دالة، ثم أخذنا تقاطع المجالين لإيجاد المجال المشترك.

(3) حللنا المقامات، واخترلنا العوامل المشتركة، فتبين أن الدالتين تبسطان إلى نفس الصورة $\dfrac{1}{x-4}$ ، ثم حددنا قيم x المستبعدة، وأخذنا تقاطع المجالين لإيجاد المجال المشترك.

10 If the following four quantities are proportional, what is the set of possible values of x?

$x,\ x^2 - 3,\ 8,\ x^2 + 3x,\ x^2 - 9$

$\underbrace{\dfrac{x}{x^2 - 3x}}_{f(x)} = \underbrace{\dfrac{x+3}{x^2 - 9}}_{g(x)}$

Proportional Quantities: $\dfrac{a}{b} = \dfrac{c}{d}$

$f(x) = \dfrac{x}{x(x-3)}$

$D_f = R - \{0, 3\}$

$g(x) = \dfrac{x+3}{(x-3)(x+3)}$

$\dfrac{x+3}{x-3}$

$D_g = R - \{-3, 3\}$

Solution

Common domain = intersection of D_f and D_g

$= R - \{-3, 0, 3\}$

شرح مبسط (بالمصطلحات الرياضية)

لأن الكميات متناسبة، كتبنا قنا بين $\frac{x}{x^2-3x} = \frac{x+3}{x^2-9}$: أول كميتين: لان = ثانى حللنا المقامات (Factorization) ثم اخترلنا العوامل المشتركة (Cancelling common factors) فتبين أن الدالتين $\frac{1}{x-3}$ ، حددنا قيم x التي تجعل المقامات = 0 (Excluded values):

$(x-3)(x+3) = 0 \Rightarrow x = \pm 3$ ومن $x(x-3) = 0 \Rightarrow x = 0, 3$

المجال المشترك = تقاطع المجالين = القيم غير المستبعدة من أي منهما:

$R - \{-3, 0, 3\}$،

Key Math Terminology Used

- Factorization: rewriting an expression as a product of factors.
- Cancelling common factors: dividing numerator and denominator by their common factor.
- Domain (Excluded values): values that make the denominator = 0.
- Common domain: intersection of the domains of the functions.
- Proportional quantities: $\frac{a}{b} = \frac{c}{d}$
- Equivalent rational functions: functions that are equal on their common domain.

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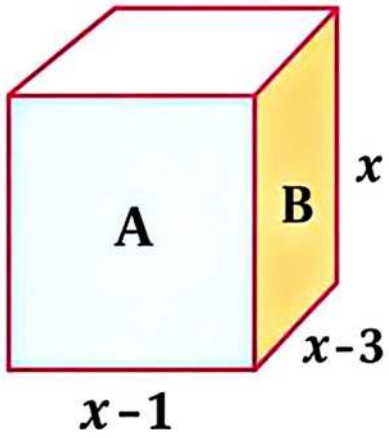
CREATIVE THINKING

LESSON 2 - 4

المجال المشترك للدوال النسبية
Common Domain of Rational Functions



11

في الشكل المقابل : صندوق على شكازي متوازي.
إذا كانت الدالة f معرفة بالنسبة بين مساحة الوجه A
إلى المساحة الجانبية، والدالة g معرفة بالنسبة بين
مساحة الوجه B والمساحة الجانبية، فاكتب كلًا من $g(x)$, $g(x)$
وحدد المجال المشترك للدالتين.

In the opposite figure: a box in the shape of a rectangular prism. If the function f is defined as the ratio of the area of face A to the lateral surface area, and the function g is defined as the ratio of the area of face B to the lateral surface area, write both $f(x)$ and $g(x)$, and find the common domain.

Lateral Surface Area

$$2(l+w)\times h = \text{Lateral Surface Area}$$

$$2(x-1+x-3)\times x =$$

$$2x(2x-4) =$$

$$4x^2-8x =$$

Function g : ratio of area of face B to lateral surface area

$$g(x)=\frac{x(x-3)}{4x^2-8x}$$

$$g(x)=\frac{x(x-3)}{4x(x-2)}$$

$$g(x)=\frac{x-3}{4(x-2)}$$

$$D_g = R-\{0,2\}$$

(Denominator $\neq 0 \Rightarrow x \neq 0, 2$)

Function f : ratio of area of face A to lateral surface area

$$f(x)=\frac{x(x-1)}{4x^2-8x}$$

$$f(x)=\frac{x(x-1)}{4x(x-2)}$$

$$f(x)=\frac{x-1}{4(x-2)}$$

$$D_f = R-\{0,2\}$$

(Denominator $\neq 0 \Rightarrow x \neq 0, 2$)

Common domain of the two functions f and g

$$R-\{0,2\}$$

Explanation (Arabic)

- المساحة الجانبية له توازي المستطيلات = $2(l+w)\times h$.
- محلنا المقامات إلى عوامل خطية (Factorization).
- اختصرنا العوامل المشتركة التي في البسط والمقام (Canceling common factors)
- حدنا المجال لكل دالة باستبعاد القيم التي تجعل المقام = 0 .
- المجال المشترك للدالتين هو تقاطع المجالين.

Key Math Terminology Used

- Factorization: writing an expression as a product of factors.
- Canceling common factors: dividing numerator and denominator by their common factor.
- Domain (Excluded values): all real numbers that make the denominator ≠ 0.
- Common domain: intersection of the domains of the functions.
- Lateral surface area of a rectangular prism = $2(l+w)\times h$.

LESSON REVIEW

العمليات على الكسور الجبرية النسبية
Operations on Rational Algebraic Fractions

LESSON 2 - 5

Lesson Evaluation

First — Measure Concepts

▸ Choose the correct answer from the given options:

(1) $\frac{2}{x-2} - \frac{x}{x-2} = ?$

Solution

$$\frac{2}{x-2} - \frac{x}{x-2} = \frac{2-x}{x-2}$$
$$= -\frac{(x-2)}{x-2}$$
$$= -1$$
$$x \neq 2$$

(A) $\frac{2-x}{2(x-2)}$ (B) $\frac{x+2}{x-2}$

✓ −1 (i)

(2) $\frac{1}{x-2} + \frac{1}{x+2} = ?$

Solution

$$\frac{1}{x-2} + \frac{1}{x+2} = \frac{(x+2)+(x-2)}{(x-2)(x+2)}$$
$$= \frac{2x}{(x-2)(x+2)}$$

(A) $\frac{2}{(x-2)(x+2)}$ (B) $\frac{1}{(x-2)(x+2)}$

(C) $\frac{2x}{(x-2)(x+2)}$ ✓ (D) $\frac{4}{(x-2)(x+2)}$

(3) Which of the following is not the additive inverse of the rational expression $\frac{x+2}{x-3}$?

Solution

The additive inverse of a rational expression is obtained by changing the sign of the expression.

$$\frac{x+2}{x-3}$$

Additive inverses:

(A) $\frac{-x-2}{x-3}$ (C) $-\frac{x+2}{x-3}$

(B) $\frac{x+2}{3-x}$ (D) $\frac{x-3}{x+2}$ ✓

(4) In which set is the reciprocal of the function $f(x) = \frac{x-2}{x+3}$ defined?

Solution

The reciprocal function is defined as long as the original function is defined and its value is not zero.

$$x+3 \neq 0 \Rightarrow x \neq -3$$
$$x-2 = 0 \Rightarrow x = 2$$

The domain is: $R - \{-3, 2\}$

(A) $R - \{-3\}$ (C) $R - \{-3, 2\}$

(B) $R - \{-3, 0\}$ (D) $R - \{2, -3\}$ ✓

(5) $\frac{x}{x-1} \times \frac{x^2-1}{x^2} = ?$

Solution

$$\frac{x}{x-1} \times \frac{x^2-1}{x^2} = \frac{x}{x-1} \times \frac{(x-1)(x+1)}{x^2}$$
$$= \frac{x+1}{x}$$

(A) $\frac{x-1}{x}$ (C) $\frac{x}{x-1}$

(B) $\frac{x-1}{x}$ (D) $\frac{x+1}{x}$ ✓

(6) In which set is the function $f(x) = \frac{x-2}{x+3} + \frac{x}{x-5}$ defined?

Solution

The domain is all real numbers except those that make any denominator equal to zero.

$$x+3 \neq 0 \Rightarrow x \neq -3 \quad \text{and} \quad x-5 \neq 0 \Rightarrow x \neq 5$$

Domain: $R - \{-3, 5\}$

(A) $R - \{-3, 0, 5\}$ (C) $R - \{-3\}$

(B) $R - \{-3, 5, 0\}$ (D) $R - \{-3, 5\}$ ✓

Explanation (Arabic)

(1) جمع وطرح كسور لها نفس المقام : نجمع أو نطرح البسطين ونبقي المقام.

(2) جمع كسور بمقامات مختلفه: نوحدالمقامات (LCM) ثمَّ نجمع.

(3) المعكوس الجمعي (Additive Inverse): تغيير إشارة المقدار.

(4) مقلوب الدالة 1 (Reciprocal) : $\frac{1}{f(x)}$ ، ويكون معرفع حيث $f(x) \neq 0$.

(5) ضرب الكسور: نضرب البسطين والمقامين ثم نبسط بالعوامل المشتركة.

(6) المجال (Domain): جميع الأعداد الحقيقية ما عدا التي تجعل المقام = 0.

Key Math Terminology Used

- Rational algebraic fractions: expressions of the form $\frac{P(x)}{Q(x)}$.
- Like denominators: same denominator.
- Common denominators (LCM): least common denominator.
- Additive inverse: for any expression E, its additive inverse is $-E$.
- Reciprocal of a function: $\frac{1}{f(x)}$, defined where $f(x) \neq 0$.
- Domain (Excluded values): all real numbers that do not make any denominator equal to zero.

LESSON REVIEW

Operations on Rational Algebraic Fractions
العمليات على الكسور الجبرية النسبية

2027 Third Prep Grade

LESSON 2 - 5

8 If $f(x) = \frac{4}{3x} + \frac{5}{3x}$, then $[f(x)]^{-1} =$ .....

A $\frac{3}{x}$

B $\frac{x}{3}$ ✓

C $\frac{6}{x}$

D $\frac{x}{6}$

Solution

$$f(x) = \frac{4}{3x} + \frac{5}{3x} = \frac{4+5}{3x} = \frac{9}{3x} = \frac{3}{x}$$

$$y = f(x) = \frac{3}{x}$$

By inverse : $x = \frac{3}{y}$

$$f^{-1}(x) = \frac{3}{x}$$

$$[f(x)]^{-1} = \frac{x}{3}$$

7 If $g(x)$ is the inverse of $f(x) = \frac{x-5}{x+7}$, and $g(k) = \frac{3}{2}$, find the value of k ?

A 2

B 3

C 29 ✓

D -29

Solution

- Since $g(x)$ is the inverse of $f(x)$, then $g(x) = \frac{1}{f(x)}$
- $g(x) = \frac{1}{f(x)} = \frac{1}{\frac{x-5}{x+7}} = \frac{x+7}{x-5}$
- $g(k) = \frac{k+7}{k-5} = \frac{3}{2}$
- $2(k+7) = 3(k-5)$
- $2k + 14 = 3k - 15$
- $2k - 3k = -15 - 14$
- $-k = -29 \Rightarrow k = 29$

Answer : $k = 29$

9 Spot the mistake in each of the following :

$\frac{x}{x+1} - \frac{5}{x+3}$ Given

$= \frac{x-5}{(x+1)(x+3)}$ Wrong step (Did not use LCM)

Correction

$\frac{x}{x+1} - \frac{5}{x+3} = \frac{x(x+3) - 5(x+1)}{(x+1)(x+3)}$

$= \frac{x^2 + 3x - 5x - 5}{(x+1)(x+3)}$

$= \frac{x^2 - 2x - 5}{(x+1)(x+3)}$

$\frac{x^2}{x-x^2} \times \frac{5x}{x^2-5x}$ Given

$= \frac{1}{x} \times \frac{1}{x^2} = \frac{1}{x^3}$ Wrong step (Cancelled incorrectly)

Correction

$\frac{x^2}{x-x^2} \times \frac{5x}{x^2-5x} = \frac{x^2}{x(1-x)} \times \frac{5x}{x(x-5)}$

$= \frac{x^2 \times 5x}{x(1-x) \times x(x-5)}$

$= \frac{5x^3}{x^2(1-x)(x-5)}$

$= \frac{5x}{(1-x)(x-5)}$

$\frac{-4}{2-x} + \frac{2x}{x-2}$ Given

$= \frac{-4-2x}{2-x} = -2$ Wrong step (Did not make common denominator)

Correction

$\frac{2x}{x-2} = \frac{-2x}{2-x}$ (Because $x - 2 = -(2 - x)$)

$\frac{-4}{2-x} + \frac{2x}{x-2} = \frac{-4}{2-x} + \frac{-2x}{2-x}$

$= \frac{-4-2x}{2-x}$

$= \frac{-2(2+x)}{2-x}$

10 Simplify each of the following expressions in simplest form (if possible) :

3 $\frac{x}{3} - \frac{4}{x}$

Solution

$= \frac{x^2 - 12}{3x}$

$= \frac{(x - 2\sqrt{3})(x + 2\sqrt{3})}{3x}$

Answer : $\frac{x^2 - 12}{3x}$

2 $\frac{2}{x-5} + \frac{x^2-27}{x-5}$

Solution

$= \frac{2 + x^2 - 27}{x-5}$

$= \frac{x^2 - 25}{x-5}$

$= \frac{(x-5)(x+5)}{x-5} = x + 5$

Answer : $x + 5$

1 $\frac{5x}{x-2} - \frac{10}{x-2}$

Solution

$= \frac{5x - 10}{x-2}$

$= \frac{5(x-2)}{x-2} = 5$

Answer : 5

شرح مبسط (بمصطلحات رياضية)

(1) جمعت أو طرحت الكسور الجبرية باستخدام المقام المشترك الأصغر (LCM).
(2) طرحت أو ضربت بعد تجويل المقام عند الضرورة.
(3) ضربت البسط والبسط والمقام بالمرافق أو عامل مشترك.
(4) وجذت مقلوب الدالة باستبدال $f(x)$ بـ $\frac{1}{f(x)}$ مع مراعاة أن المقام $\neq 0$.
(5) لإيجاد المعكوس الجبري (Additive inverse) غيرت إشارة الكسر.
(6) بسطت الناتج بإلغاء العوامل المشتركة وكتابة أبسط صورة ممكنة.

Key Math Terminology Used

- Rational algebraic fractions (RAF): expressions of the form $\frac{P(x)}{Q(x)}$.
- Like denominators: same denominator.
- Least common denominator (LCD): least common denominator.
- Reciprocal (multiplicative inverse): $\frac{1}{f(x)}$, defined where $f(x) \neq 0$.
- Additive inverse: for any expression E, its additive inverse is $-E$.
- Domain (excluded values): all real numbers that do not make the denominator equal to zero.

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D.M.RAZK موقع الدكتور محمد رزق معلم الكيمياء التعليمي

LESSON REVIEW

Operations on Rational Algebraic Fractions

العمليات على الكسور الجبرية النسبية

LESSON 2 - 5

4 $\frac{1}{x-3} + \frac{1}{x-4}$

Solution

$= \frac{x-4+x-3}{(x-3)(x-4)}$

$= \frac{2x-7}{(x-3)(x-4)}$

Answer: $\frac{2x-7}{(x-3)(x-4)}$

Domain: $x \neq 3, 4$

5 $\frac{x}{x+3} \div \frac{x^2}{x^2+3x}$

Solution

$\frac{x}{x+3} \times \frac{x^2+3x}{x^2}$

$= \frac{x}{x+3} \times \frac{x(x+3)}{x^2}$

$= 1$

Answer: 1

Domain: $x \neq 0, -3$

6 $\frac{3x-6}{5x-20} \times \frac{10x-40}{27x-54}$

Solution

$\frac{3(x-2)}{5(x-4)} \times \frac{10(x-4)}{27(x-2)}$

$= \frac{3 \times 10}{5 \times 27} = \frac{30}{135} = \frac{2}{9}$

Answer: $-\frac{2}{9}$

Domain: $x \neq 4, 2$

11 أوجد $f(x)$ في أبسط صورة موضحا مجال f في كل سؤال:

Find $f(x)$ in simplest form and state the domain in each question:

1 $f(x) = \frac{x^2+x}{x^2+3x+2} + \frac{2x}{x^2+2x}$

Solution

$= \frac{x(x+1)}{(x+1)(x+2)} + \frac{2x}{x(x+2)}$

$= \frac{x}{x+2} + \frac{2}{x+2} = \frac{x+2}{x+2} = 1$

Answer: 1 Domain: $x \neq 0, -1$

2 $f(x) = \frac{x^2+6x}{x^2+5x} + \frac{x^2+2x-8}{x^2+3x-10}$

Solution

$= \frac{x(x+6)}{x(x+5)} + \frac{(x+4)(x-2)}{(x+5)(x-2)}$

$= \frac{x+6}{x+5} + \frac{x+4}{x+5} = \frac{(x+6)+(x+4)}{x+5}$

$= \frac{2x+10}{x+5} = \frac{2(x+5)}{x+5} = 2$

Answer: 2 Domain: $x \neq 0, -5$

3 $f(x) = \frac{1}{x-4} - \frac{8}{x^2-16}$

Solution

$= \frac{1}{x-4} - \frac{8}{(x-4)(x+4)}$

$= \frac{x+4}{(x-4)(x+4)} = \frac{8}{(x-4)(x+4)}$

$= \frac{x+4-8}{(x-4)(x+4)} = \frac{x-4}{(x-4)(x+4)}$

$= \frac{1}{x+4}$

Answer: $\frac{1}{x+4}$ Domain: $x \neq -4, 4$

4 $f(x) = \frac{x^2-3x+2}{x^2-4} - \frac{x^2-3x}{x^2-x-6}$

$= \frac{(x-1)(x-2)}{(x-2)(x+2)} - \frac{x(x-3)}{(x-3)(x+2)}$

$= \frac{x-1}{x+2} = \frac{x}{x+2} = \frac{(x-1)-x}{x+2}$

$= \frac{-1}{x+2}$

Answer: $-\frac{-1}{x+2}$ Domain: $x \neq -2, 2, 3$

10 Simplify each of the following expressions in simplest form (if possible):

1 $\frac{5x}{x-2} - \frac{10}{x-2}$

Solution

$= \frac{5x-10}{x-2}$

$= \frac{5(x-2)}{x-2} = 5$

Answer: 5

Domain: $x \neq 2$

2 $\frac{2}{x-5} + \frac{x^2-27}{x-5}$

Solution

$= \frac{2+x^2-27}{x-5}$

$= \frac{x^2-25}{x-5}$

$= \frac{(x-5)(x+5)}{x-5} = x+5$

Answer: $x+5$

Domain: $x \neq 5$

3 $\frac{x}{3} - \frac{4}{x}$

Solution

$= \frac{x^2-12}{3x}$

$= \frac{(x-2\sqrt{3})(x+2\sqrt{3})}{3x}$

Answer: $\frac{x^2 \quad 12}{3x}$

Domain: $x \neq 0$

شرح مبسط (بمصطلحات رياضية)

(1) في (4, 5, 6): حللت البسط والمقام إلى عوامل (Factorization)، ثم اختصرت العوامل المشتركة، وجمعت أو طرحت أو ضربت أو قسمت.

(2) في (11): وضعت المقامات على صورة العوامل لتحديد الـ LCD، ثم كتبت الكسور بمقامات مشتركة، وجمعت أو طرحت البسطين.

(3) في (10): وحدت المقامات، ثم بسطت والبسط بند إذا أمكن، واختصرت العوامل المشتركة.

(4) المجال (Domain): جميع قيم x الحقيقية ما عدا القيم التي تجعل أي مقام = 0.

Key Math Terminology Used

- Rational algebraic fractions: expressions of the form $\frac{P(x)}{Q(x)}$.
- Factorization: writing an expression as a product of factors.
- Cancelling common factors: simplifying by dividing numerator and denominator by their common factor.
- LCD (Least common denominator): the least common denominator.
- Add fractions: write with a common denominator, then add numerators.
- Subtract fractions: write with a common denominator, then subtract numerators.
- Multiply: multiply numerators and denominators.
- Divide: multiply by the reciprocal (multiplicative inverse).
- Domain (Excluded values): all real numbers that do not make the denominator equal to zero.

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LESSON REVIEW

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العمليات على الكسور الجبرية النسبية

LESSON 2 - 5

6 $f(x) = \dfrac{x^2+2x+4}{x^3-8} + \dfrac{x^2-x-12}{x^2+x-6}$

Solution

$x^3-8 = (x-2)(x^2+2x+4)$

$x^2+x-6 = (x+3)(x-2)$

$f(x) = \dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \dfrac{x^2-x-12}{(x+3)(x-2)}$

$= \dfrac{1}{x-2} + \dfrac{(x-4)(x+3)}{(x+3)(x-2)}$

$= \dfrac{1}{x-2} + \dfrac{x-4}{x-2} = \dfrac{1+x-4}{x-2} = \dfrac{x-3}{x-2}$

Answer : $\dfrac{x-3}{x-2}$, $x \neq 2, -3$

5 $f(x) = \dfrac{3x+15}{x^2+7x+10} - \dfrac{x+5}{x^2+6x+5}$

Solution

$x^2+7x+10 = (x+5)(x+2)$

$x^2+6x+5 = (x+5)(x+1)$

$f(x) = \dfrac{3(x+5)}{(x+5)(x+2)} - \dfrac{x+5}{(x+5)(x+1)}$

$= \dfrac{3}{x+2} - \dfrac{1}{x+1} = \dfrac{3(x+1)-(x+2)}{(x+2)(x+1)}$

$= \dfrac{3x+3-x-2}{(x+2)(x+1)} = \dfrac{2x+1}{(x+2)(x+1)}$

Answer : $\dfrac{2x+1}{(x+2)(x+1)}$, $x \neq -2, -1$

8 $f(x) = \dfrac{x^2-11x+28}{x^2-13x+42} \times \dfrac{x^2+7x+10}{20-x-x^2}$

Solution

$x^2-11x+28 = (x-7)(x-4)$

$x^2-13x+42 = (x-6)(x-7)$

$x^2+7x+10 = (x+5)(x+2)$

$20-x-x^2 = -(x^2+x-20) = -(x+5)(x-4)$

$f(x) = \dfrac{(x-7)(x-4)}{(x-6)(x-7)} \times \dfrac{(x+5)(x+2)}{-(x+5)(x-4)}$

$= \dfrac{x+2}{-(x-6)} = -\dfrac{x+2}{x-6}$

Answer : $-\dfrac{x+2}{x-6}$, $x \neq 7, 6, -5, 4$

7 $f(x) = \dfrac{x^2+x-20}{x^2+2x-15} \times \dfrac{x^2+4x-21}{x^2+3x-28}$

Solution

$x^2+x-20 = (x+5)(x-4)$

$x^2+2x-15 = (x+5)(x-3)$

$x^2+4x-21 = (x+7)(x-3)$

$x^2+3x-28 = (x+7)(x-4)$

$f(x) = \dfrac{(x+5)(x-4)}{(x+5)(x-3)} \times \dfrac{(x+7)(x-3)}{(x+7)(x-4)}$

$= 1$

Answer : 1 , $x \neq -5, 3, -7, 4$

10 $f(x) = \dfrac{3x-15}{x+4} \div \dfrac{5-x}{6x+24}$

Solution

$= \dfrac{3x-15}{x+4} \times \dfrac{6x+24}{5-x}$

$3x-15 = 3(x-5)$, $6x+24 = 6(x+4)$, $5-x = -(x-5)$

$f(x) = \dfrac{3(x-5)}{x+4} \times \dfrac{6(x+4)}{-(x-5)}$

$= -18$

Answer : -18 , $x \neq -4, 5$

9 $f(x) = \dfrac{2x^2+5x+2}{2x^2+7x+3} \times \dfrac{x^2-7x-30}{x^2-6x-40}$

Solution

$2x^2+5x+2 = (2x+1)(x+2)$

$2x^2+7x+3 = (2x+1)(x+3)$

$x^2-7x-30 = (x-10)(x+3)$

$x^2-6x-40 = (x-10)(x+4)$

$f(x) = \dfrac{(2x+1)(x+2)}{(2x+1)(x+3)} \times \dfrac{(x-10)(x+3)}{(x-10)(x+4)}$

$= \dfrac{x+2}{x+4}$

Answer : $\dfrac{x+2}{x+4}$, $x \neq -\dfrac{1}{2}, -3, 10, -4$

شرح مبسط (بمصطلحات رياضية)

(1) (تحليل Factorization) : حللت كل كثير حدود إلى عوامل.

(2) (تبسيط) : اختصرت العوامل المشتركة في البسط والمقام.

(3) (المشترك الأصغر) : جمعت أو طرحت الكسور بعد توحيد المقامات.

(4) Multiplication (الضرب) : ضربت الكسور بضرب البسط × البسط والمقام × المقام.

(5) (القسمة) Divisiti : حولت القسمة إلى ضرب في المقلوب.

(6) (المجال) Do : استبعدت القيم التي تجعل أي مقام = 0 .

Key Math Terminology Used

- Rational algebraic fractions (RAF): expressions of the form $\frac{P(x)}{Q(x)}$.
- Factorization: writing an expression as a product of factors.
- Cancellation: simplifying by dividing numerator and denominator by their common factor.
- LCD (least common denominator): the least common denominator.
- Addition / Subtraction: write with a common denominator, then add or subtract.
- Multiplication: multiply numerators and denominators.
- Division: multiply by the reciprocal (multiplicative inverse).
- Domain (Excluded values): all real numbers that do not make the denominator equal to zero.

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LESSON REVIEW

Operations on Rational Algebraic Fractions

العمليات على الكسور الجبرية النسبية

LESSON 2 - 5

12 Find $f(x) = \dfrac{6x^2+13x+6}{4x^2-9} \div \dfrac{6x^2+x-2}{4x^2-1}$ in simplest form.

Model Solution

$$f(x) = \frac{6x^2+13x+6}{4x^2-9} \times \frac{4x^2-1}{6x^2+x-2}$$

$$= \frac{(3x+2)(2x+3)}{(2x-3)(2x+3)} \times \frac{(2x-1)(2x+1)}{(3x+2)(2x-1)}$$

$$= \frac{2x+1}{2x-3}$$

Steps of the Solution

1. Factor the numerator and denominator.
2. Change the division to multiplication.
3. Cancel the common factors.
4. Write the simplified form.

Domain: $x \neq \pm\frac{3}{2}, -1, \frac{1}{2}$ (the values that make any original denominator equal to zero)

11 Find $f(x) = \dfrac{x^2-2x}{2x^2-x-6} \div \dfrac{2x^2-3x}{4x^2-9}$ in simplest form.

Model Solution

$$f(x) = \frac{x^2-2x}{2x^2-x-6} \times \frac{4x^2-9}{2x^2-3x}$$

$$= \frac{x(x-2)}{(2x+3)(x-2)} \times \frac{(2x-3)(2x+3)}{x(2x-3)}$$

$$= 1$$

Notes

- Factor every numerator and denominator.
- Change the division to multiplication.
- Cancel the common factors.
- Make sure no denominator equals zero.

Domain: $x \neq -\frac{3}{2}, \frac{3}{2}, 0$ (the values that make any original denominator equal to zero)

12 Find $f(x) = \dfrac{x^2-x-20}{x^2+2x-8} + \dfrac{3x+6}{x^2-4}$ in simplest form, then find $f(5)$, $f(-2)$ if possible.

Model Solution

1 Factorization

$$x^2-x-20 = (x-5)(x+4)$$
$$x^2+2x-8 = (x+4)(x-2)$$
$$3x+6 = 3(x+2)$$
$$x^2-4 = (x-2)(x+2)$$

2 Simplify (find a common denominator)

$$f(x) = \frac{(x-5)(x+4)}{(x+4)(x-2)} + \frac{3(x+2)}{(x-2)(x+2)}$$

$$= \frac{x-5}{x-2} + \frac{3}{x-2} = \frac{x-5+3}{x-2} = \frac{x-2}{x-2} = \boxed{1}$$

3 Domain of the function

$$x+4 \neq 0,\ x-2 \neq 0,\ x+2 \neq 0$$
$$\Rightarrow\ x \neq -4,\ 2,\ -2$$

Domain: $\mathrm{R} - \{-4, -2, 2\}$

4 Find the required values

$$f(5) = \frac{5-2}{5-2} = \frac{3}{3} = 1$$

$f(5) = 1$

$$f(-2) = \frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$$

$f(-2) = 1$

شرح مبسط (بمصطلحات رياضية)

(1) تحليل الحدود (Factorization): حولت كل حدود كنركثيرة إلي عوامل.

(2) القسمة إلي ضرب (Division to multiplication): قلبت الكسر الثاني وضربته في الأول.

(3) اختصار العوامل المشتركة (Cancel common factors): اختصرت العوامل المتشابهة في البسط والمقام.

(4) كتابة الصورة المبسطة (Simplify): كتبت الناتج في أبسط صورة.

(5) إيجاد المجال (Domain): استبعدت قيم x التي تجعل أي مقام = 0.

(6) التعويض (Substitution): عوضت x بالقيم المعطاة لإيجاد $f(x)$.

Key Math Terminology Used

- Rational algebraic fractions (RAF): expressions of the form $\frac{P(x)}{Q(x)}$.
- Factorization: writing an expression as a product of factors.
- Division to multiplication: multiply by the reciprocal.
- Cancel common factors: simplify by dividing numerator and denominator by their common factor.
- Common denominator (LCD): the least common denominator.
- Domain (Excluded values): all real numbers that do not make any denominator equal to zero.
- Substitution: replacing the variable with a given value.

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LESSON REVIEW

Operations on Rational Algebraic Fractions

العمليات على الكسور الجبرية النسبية

LESSON 2 - 5

Creative Thinking

13 Find $f(x)$ in simplest form where

$$f(x) = \frac{x^2 - 4}{x^2 - x - 2} \div \frac{x^2 + 5x + 6}{x^2 + 2x - 3}$$

Then find $f(3)$, $f(1)$, $f(-4)$, and if $f(k) = \frac{3}{4}$, find the value of k.

Solution

$$f(x) = \frac{x^2 - 4}{x^2 - x - 2} \div \frac{x^2 + 5x + 6}{x^2 + 2x - 3}$$

$$= \frac{(x-2)(x+2)}{(x-2)(x+1)} \div \frac{(x+2)(x+3)}{(x-1)(x+3)}$$

$$= \frac{(x-2)(x+2)}{(x-2)(x+1)} \times \frac{(x-1)(x+3)}{(x+2)(x+3)}$$

$$= \boxed{\frac{x-1}{x+1}}$$

Domain of f

Values that make the original denominators = 0.

$(x-2)(x+1)(x+2)(x+3) \neq 0$

$\Rightarrow \quad x \neq 2, -1, -2, -3$

Find the values

$f(-4) = \frac{-4-1}{-4+1} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$ | $f(1) = \frac{1-1}{1+1} = \frac{0}{2} = \boxed{0}$ | $f(3) = \frac{3-1}{3+1} = \frac{2}{4} = \boxed{\frac{1}{2}}$

Find k if $f(k) = \frac{3}{4}$

$$\frac{k-1}{k+1} = \frac{3}{4} \quad \Rightarrow \quad 4(k-1) = 3(k+1) \quad \Rightarrow \quad 4k - 4 = 3k + 3 \quad \Rightarrow \quad \boxed{k = 7}$$

14 The area of a square is $\frac{x^2 - 3x - 4}{x^2 - x - 2}$ square units, and its height is $\frac{x^2 - 7x + 12}{x^2 - 2x}$ units.

Find a rule for the length of its base in simplest form.

Solution

$$\text{Base} = \frac{\text{Area}}{\text{Height}}$$

$$\text{القاعدة} = \frac{\text{المساحة}}{\text{الارتفاع}}$$

$$\text{Base} = \frac{x^2 - 3x - 4}{x^2 - x - 2} \div \frac{x^2 - 7x + 12}{x^2 - 2x}$$

$$= \frac{x^2 - 3x - 4}{x^2 - x - 2} \times \frac{x^2 - 2x}{x^2 - 7x + 12}$$

$$= \frac{(x-4)(x+1)}{(x-2)(x+1)} \times \frac{x(x-2)}{(x-3)(x-4)}$$

$$= \boxed{\frac{x}{x-3}}$$

Notes

In the domain, the denominators must not be zero.

$x \neq 2, 0, 3, 4$

Therefore, the domain is:

$R - \{0, 2, 3, 4\}$

شرح مبسط (بمصطلحات رياضية)

(1) تحليل (Factorization): حللت كل كثير حدود إلى عوامل.

(2) تجويل القسمة إلى ضرب (Division to multiplication): قلبت الكسر الثاني وضربته في الأول.

(3) اختصار العوامل المشتركة (Cancel common factors): اختصرت العوامل المتماثلة في البسط والمقام.

(4) تبسيط (Simplify): كتبت الناتج في أبسط صورة.

(5) المجال (Domain): استبعدت القيم التي تجعل أي مقام = 0.

(6) التعويض (Substitution): عوضت القيم المعطاة لإيجاد قيم الدالة.

Key Math Terminology Used

- Rational algebraic fractions (RAF): expressions of the form $\frac{P(x)}{Q(x)}$.
- Factorization: writing an expression as a product of factors.
- Division to multiplication: multiply by the reciprocal.
- Cancel common factors: simplify by dividing numerator and denominator by their common factor.
- Domain (Excluded values): all real numbers that do not make any denominator equal to zero.
- Substitution: replacing the variable with a given value.

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العمليات على الكسور الجبرية النسبية

LESSON 2 - 5

15 Find the value of $a + b$: $\frac{x^2 + ax - b}{x^2 - 2x - 15} \times \frac{x^2 - 9x + 20}{x^2 - 16} = \frac{x - 7}{x + 3}$ if possible.

Model Solution

$$\frac{x^2 + ax - b}{x^2 - 2x - 15} \times \frac{x^2 - 9x + 20}{x^2 - 16} = \frac{x - 7}{x + 3}$$

$$= \frac{x^2 + ax - b}{(x - 5)(x + 3)} \times \frac{(x - 5)(x - 4)}{(x - 4)(x + 4)} = \frac{x - 7}{x + 3}$$

$$= \frac{\cancel{(x-5)}(x + 3)}{\cancel{(x-5)}(x + 3)} \times \frac{\cancel{(x-4)}(x + 4)}{\cancel{(x-4)}(x + 4)} = \frac{x - 7}{x + 3}$$

$$= \frac{x^2 + ax - b}{x + 3} \times \frac{1}{x + 4} = \frac{x - 7}{x + 3}$$

$$\frac{x^2 + ax - b}{x + 3} = \frac{(x - 7)(x + 4)}{x + 3}$$

$$x^2 + ax - b = (x - 7)(x + 4) = x^2 - 3x - 28$$

Comparing coefficients:

$a = -3$ $\quad$ $b = 28$ $\quad$ $a + b = -3 + 28 =$ **25**

Steps of the Solution

1. Factor the numerator and denominator.
2. Cancel the common factors.
3. Multiply by the reciprocal.
4. Simplify.
5. Since the two sides are equal, compare the coefficients.

16 If the domain of $f(x) = \frac{a}{x} + \frac{1}{b - x}$ is $R - \{0, 4\}$ and $f(2) = 3$, find the values of $a \cdot b$.

Model Solution

1 Domain of the function

Domain is $R - \{0, 4\}$

This means the denominators do not equal zero at $x = 0$ and $x = 4$.

So, $b - x = 0$ when $x = 4$

$b = 4$

2 Use $f(2) = 3$ to find a

$$f(2) = \frac{a}{x} + \frac{1}{b - x} = 3$$

Substitute $x = 2$, $b = 4$

$$f(2) = \frac{a}{2} + \frac{1}{4 - 2} = 3$$

$$\frac{a}{2} + \frac{1}{2} = 3 \Rightarrow \frac{a + 1}{2} = 3 \Rightarrow a + 1 = 6 \Rightarrow a = 5$$

Therefore: $a = 5$, $b = 4$ $\quad$ So, $a \cdot b = 5 \times 4 =$ **20**

شرح مبسط (بمصطلحات رياضية)

(1) تحليل الحدود (Factorization): تحللت كاء كتبر حدود إلي عوامل.

(2) الضرب في المعكوس (Multiplication by the reciprocal): حولت القسمة إلى ضرب في مقلوب الكسر التاني.

(3) اختصار العوامل المشتركة (Cancel common factors): حذفت العوامل المتشابهة في البسط والمقام.

(4) تبسيط (Simplify): كتبت الناتج في أبسط صورة.

(5) مجال الدالة الدانل (Domain): استبعدت القيم التي تجعل أي مقام = 0.

(6) التعويض (Substitution): عوضت قيمة العدد في المعادلة.

Key Math Terminology Used

- Rational algebraic fractions (RAF): expressions of the form $\frac{P(x)}{Q(x)}$.
- Factorization: writing an expression as a product of factors.
- Multiplication by the reciprocal: change division to multiplication by the reciprocal of the divisor.
- Cancel common factors: simplify by dividing numerator and denominator by their common factor.
- Simplify: write the result in simplest form.
- Domain (Excluded values): all real numbers that do not make any denominator equal to zero.
- Substitution: replacing the variable with a given value.

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امسح الباركود بالموبيل لتثبيت التطبيق وتحميل كل الملازم مجانا وبسرعه

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